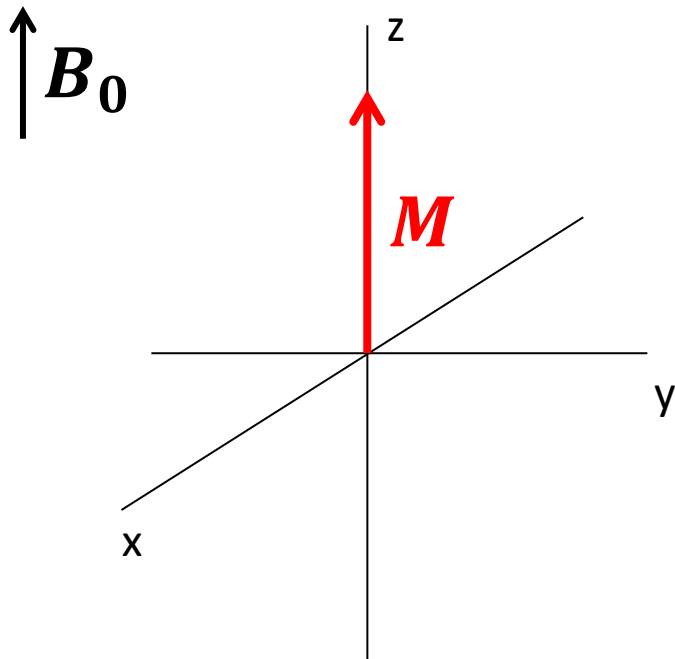
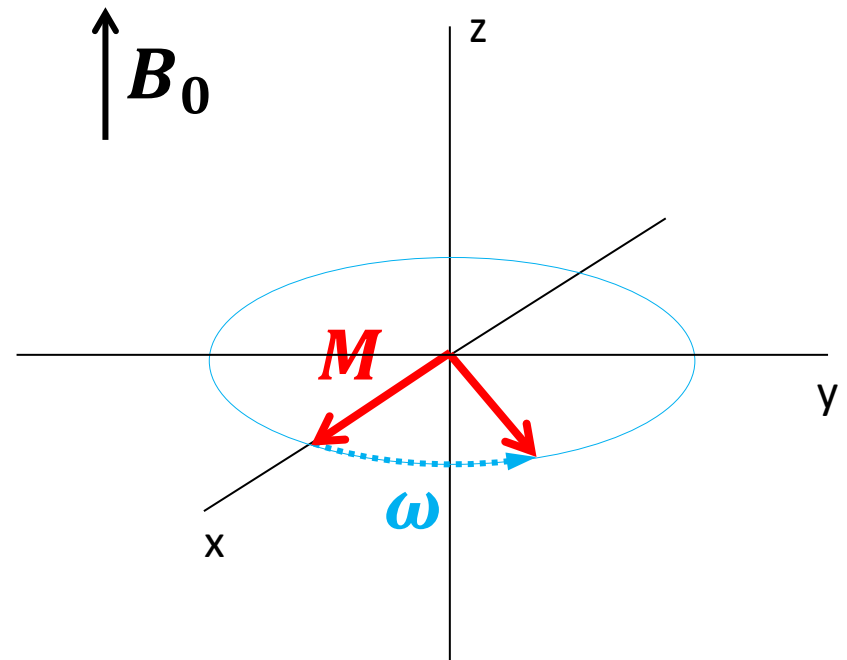


Magnetization at equilibrium is along z

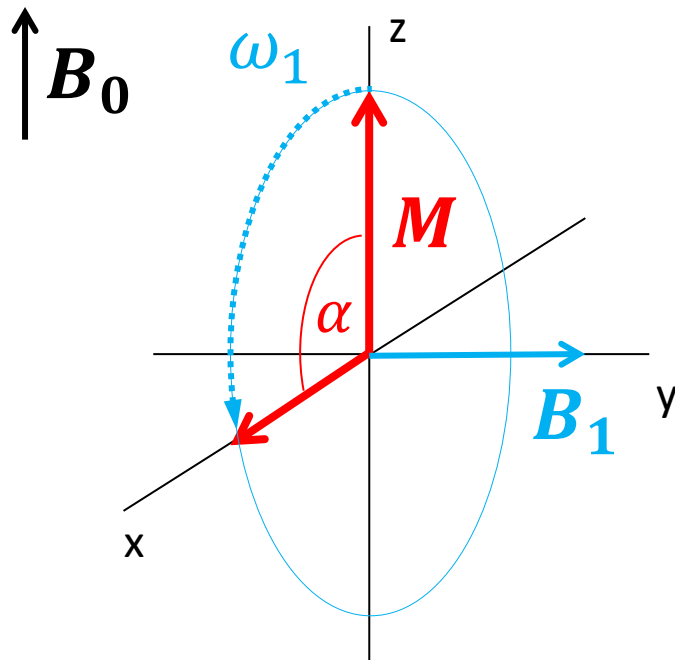


Magnetization in the x-y plane precesses with frequency $\omega = -\gamma B$ (Larmor frequency)



Magnetization is rotated from z axis to the x-y plane (or vice versa) by a pulse of perpendicular (horizontal) magnetic field B_1 that oscillates with angular velocity $\omega_{rf} \approx \omega$ (Larmor frequency)

Rotating frame (angular velocity ω_{rf})

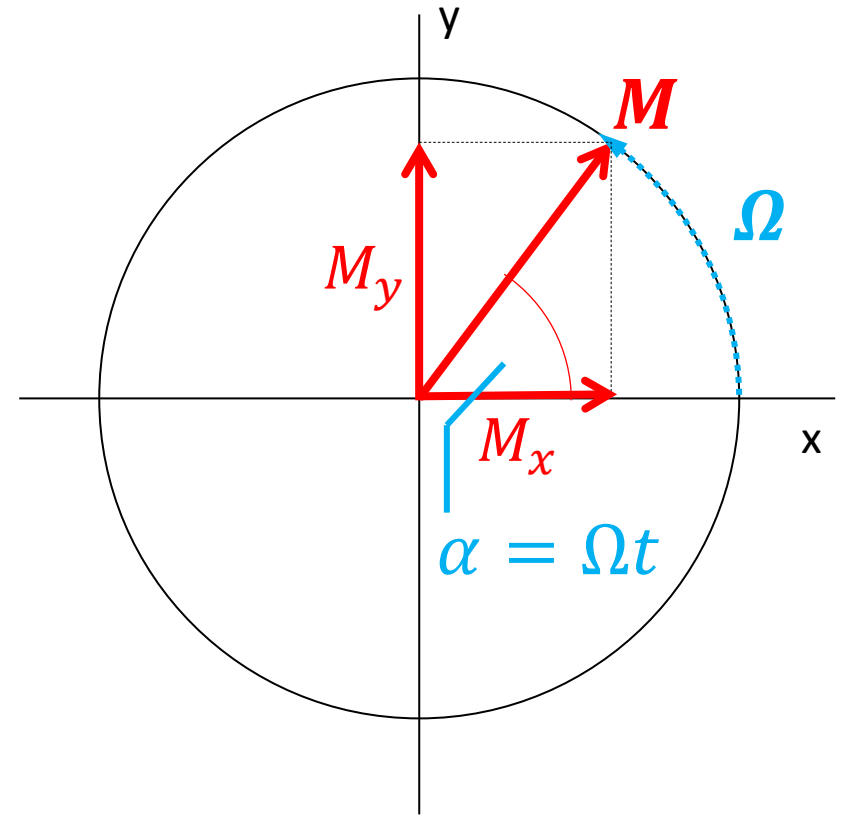
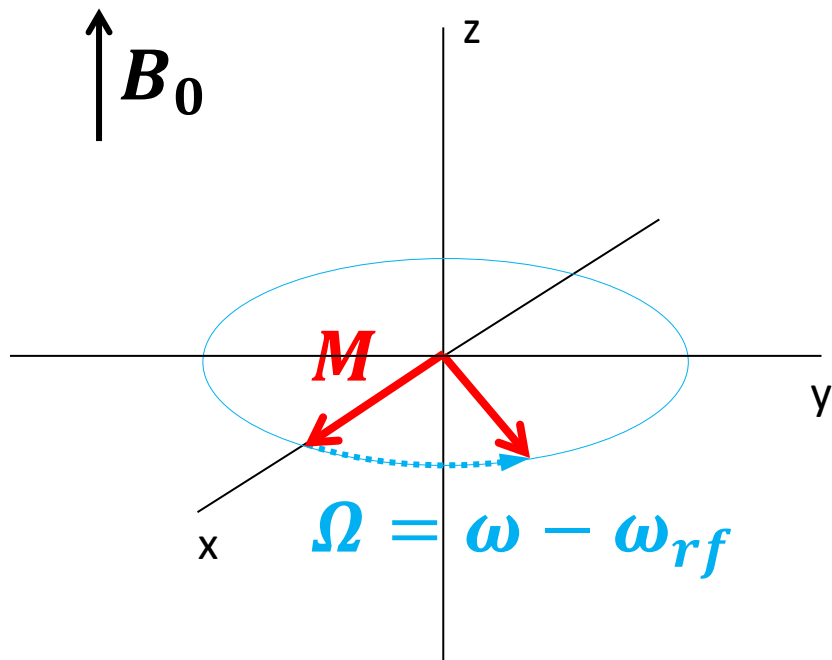


90° pulse:

Turn on B_1 for time t such that $\alpha = \omega_1 t = |-\gamma B_1| t = \frac{\pi}{2}$

Rotating frame (angular velocity ω_{rf})

Magnetization in the x-y plane precesses with frequency Ω

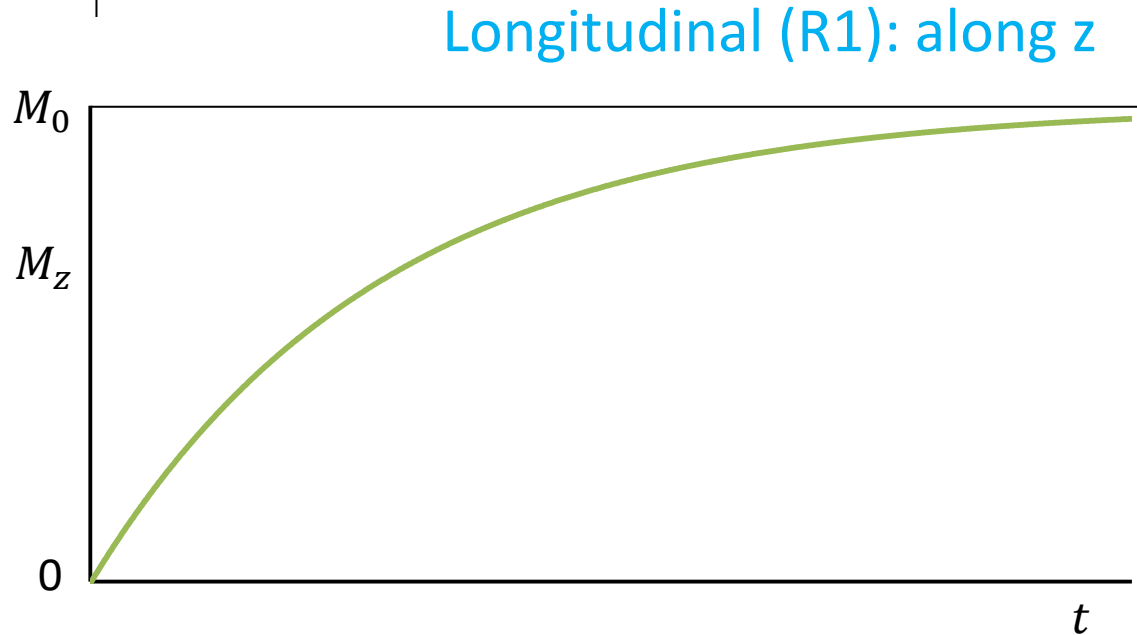
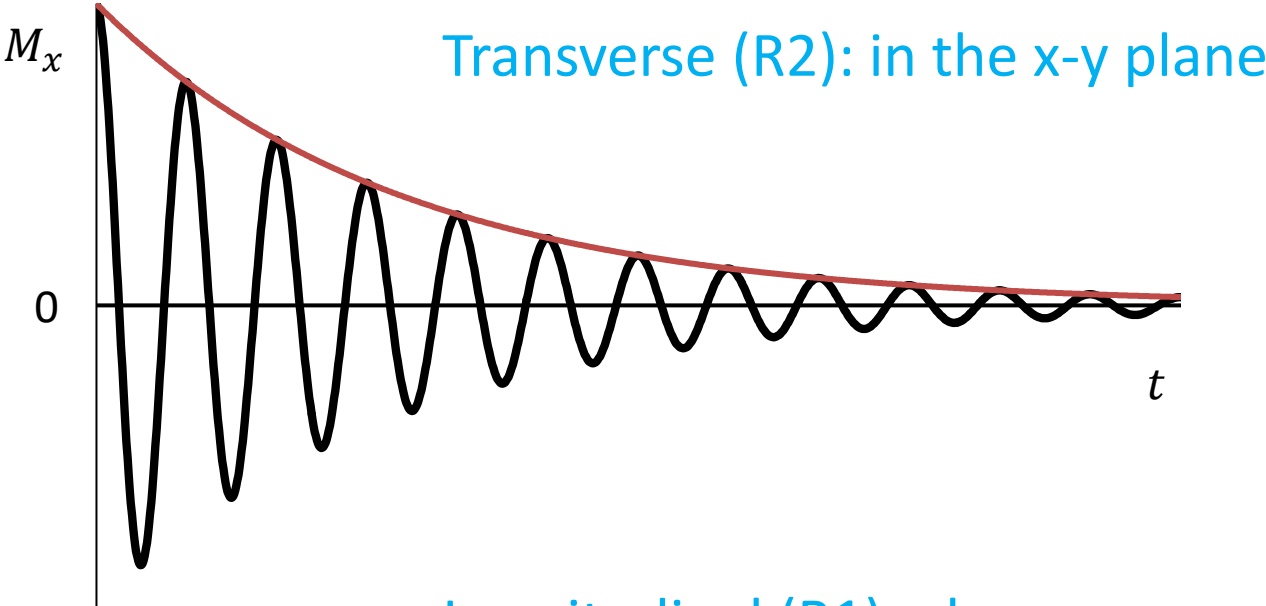


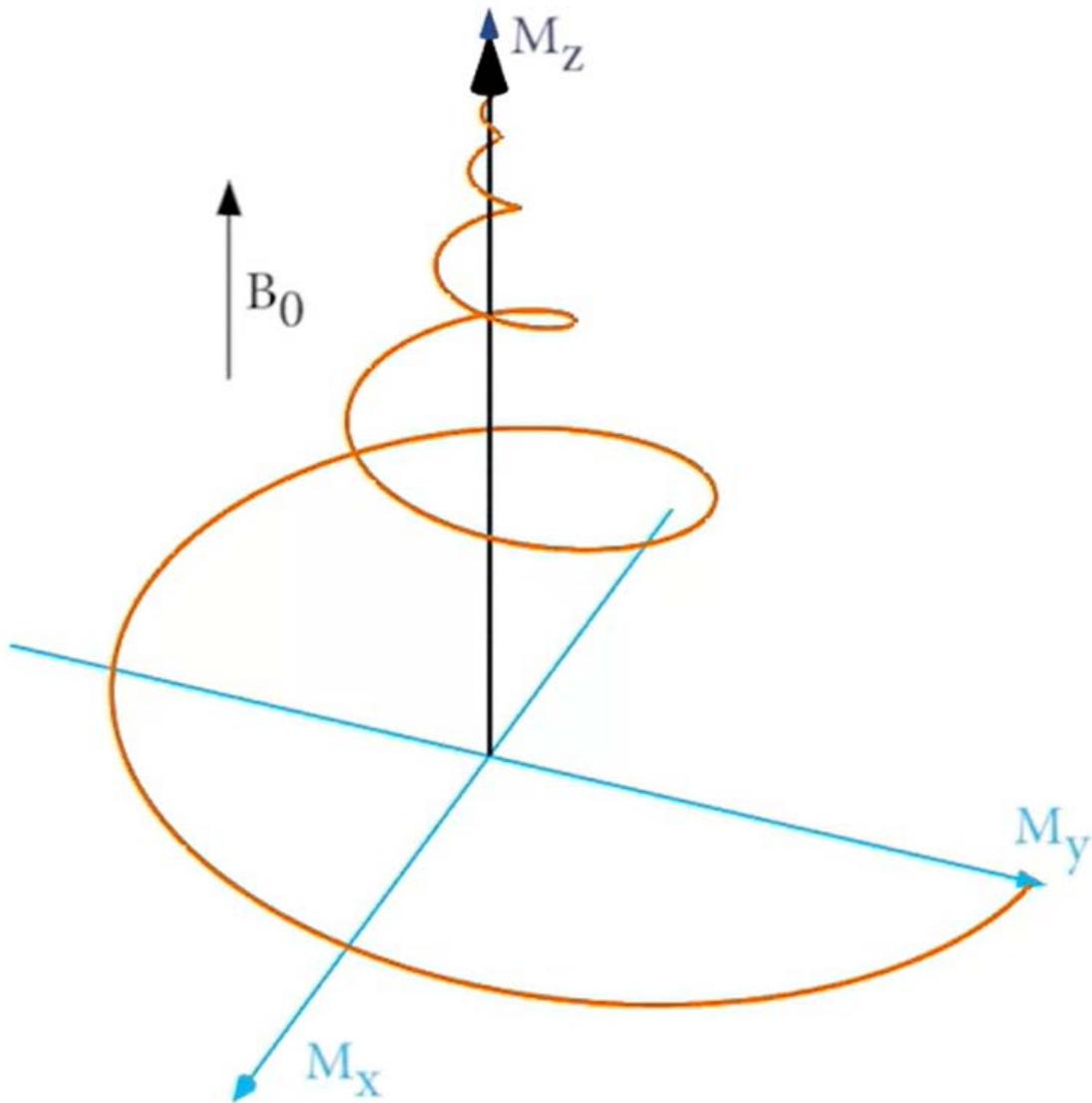
$$M_x = M_0 \cos \Omega t$$

$$M_y = M_0 \sin \Omega t$$

$$M_{xy} = M_x + iM_y = M_0 e^{i\Omega t}$$

Relaxation





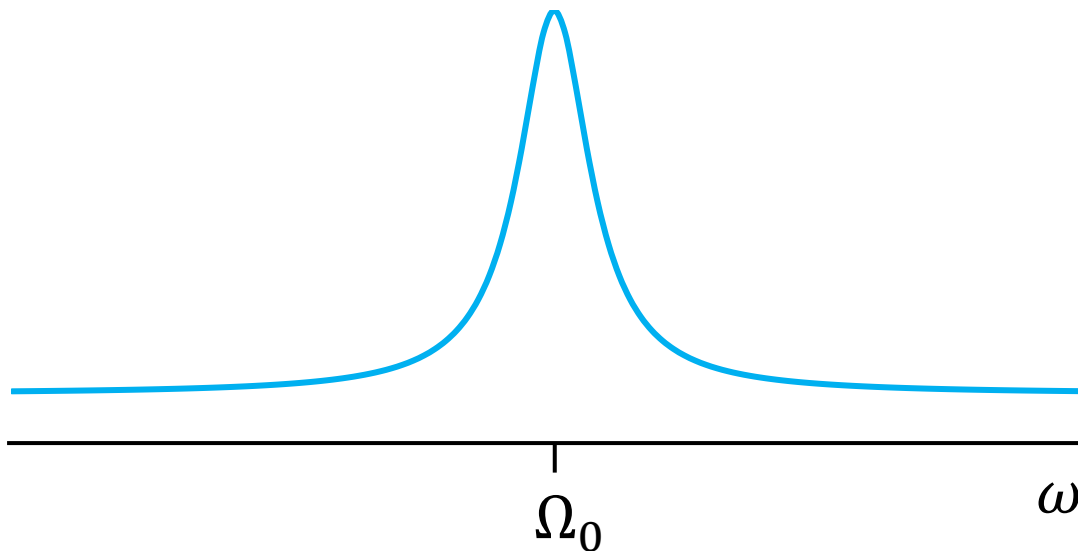
[https://en.wikipedia.org/wiki/Relaxation_\(NMR\)](https://en.wikipedia.org/wiki/Relaxation_(NMR))

Oscillation $s(t) = M_0 e^{-R_2 t} e^{i\Omega_0 t}$



Fourier transform

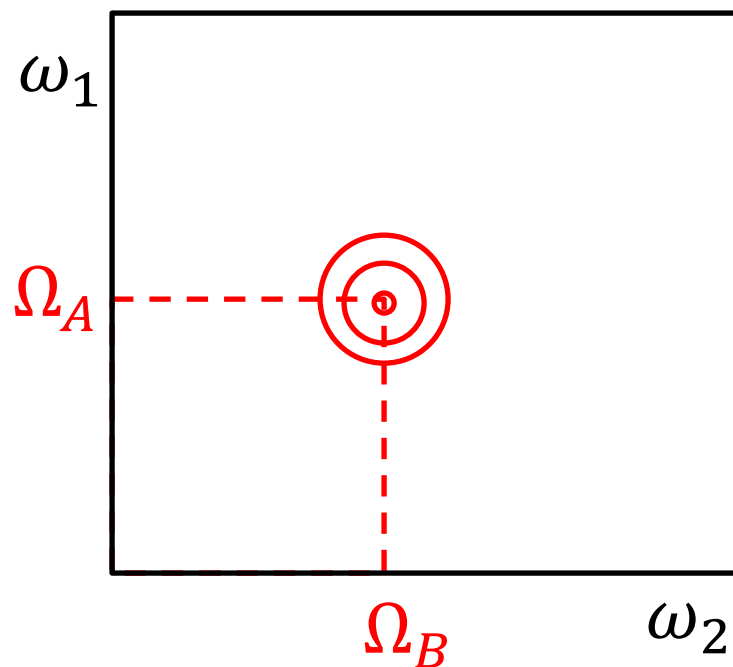
$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt$$



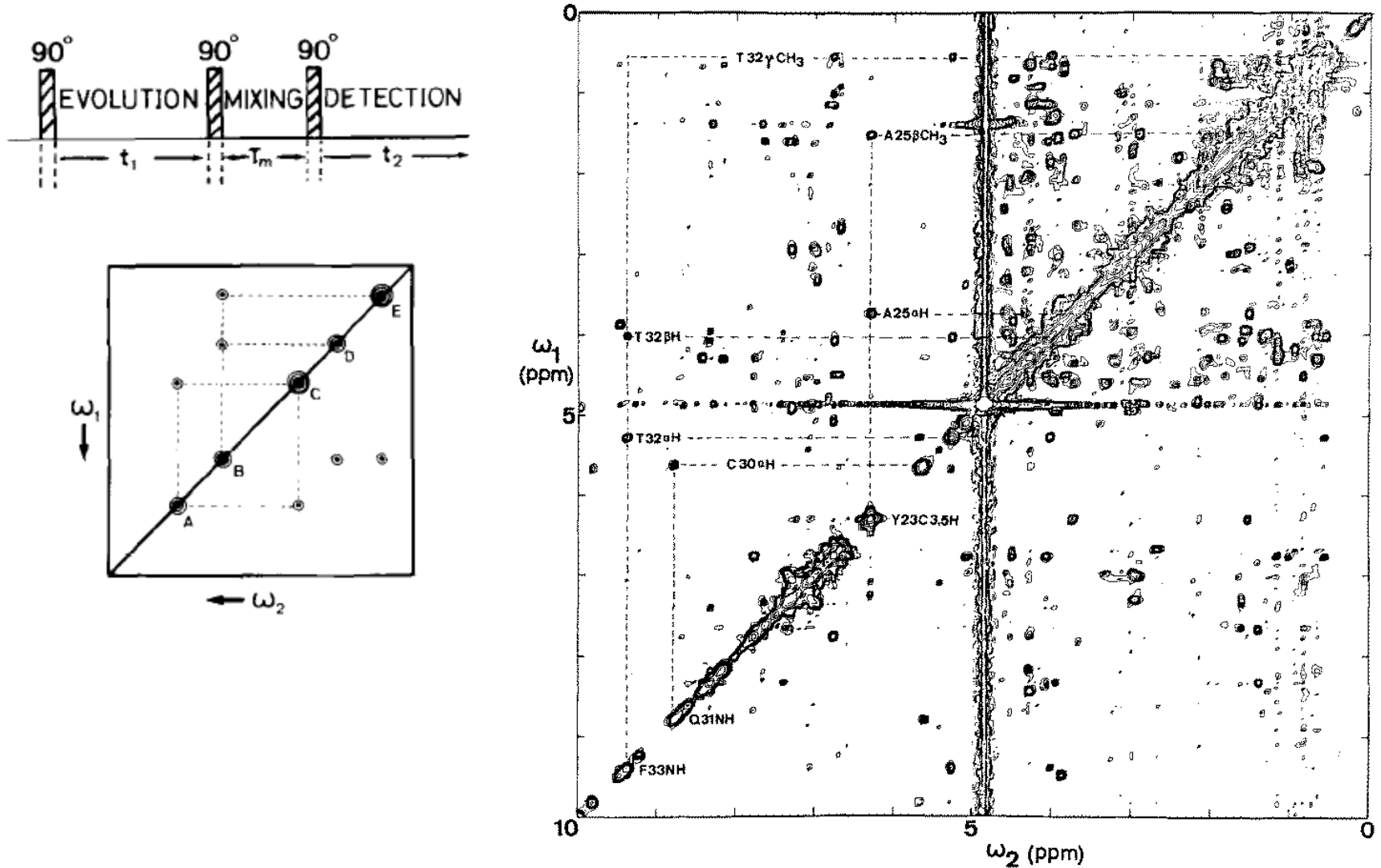
2-dimensional (2D) NMR spectroscopy

Correlation between nuclear dipoles

- Through-bond interactions (J-coupling) < 4 bonds apart
- Through-space interactions (nuclear Overhauser effect, NOE) < 5 Å apart

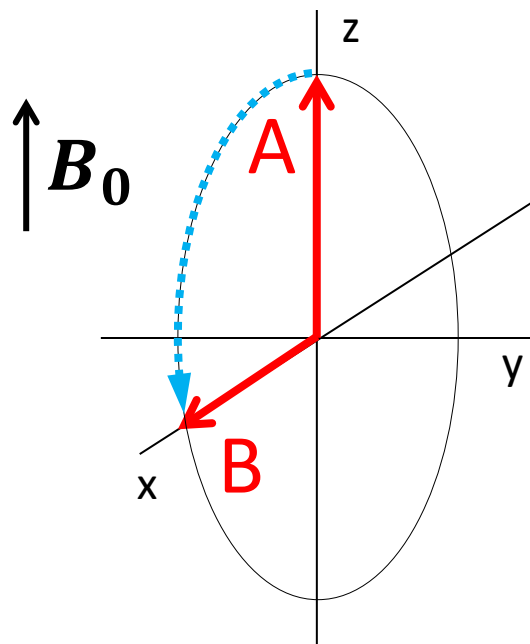
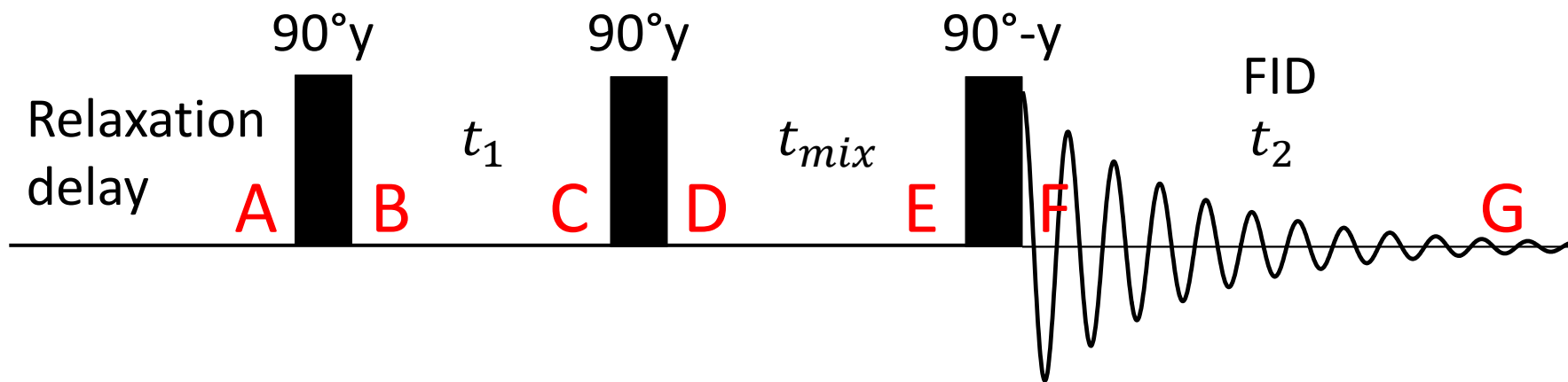


The first 2D spectrum of a protein (BPTI)



Kumar A, Ernst RR, Wüthrich K. A two-dimensional nuclear Overhauser enhancement (2D NOE) experiment for the elucidation of complete proton-proton cross-relaxation networks in biological macromolecules. *Biochem Biophys Res Commun.* 1980 Jul 16;95(1):1-6.

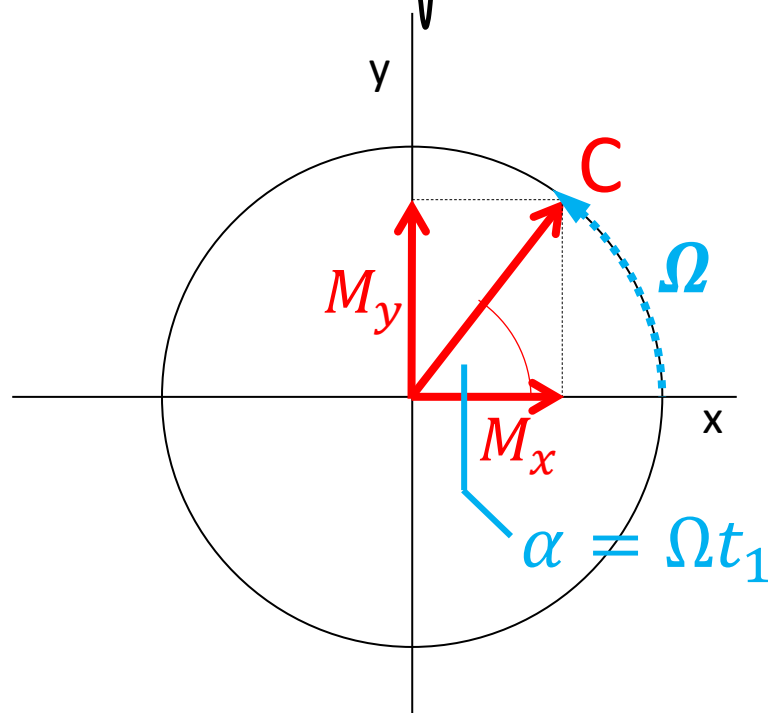
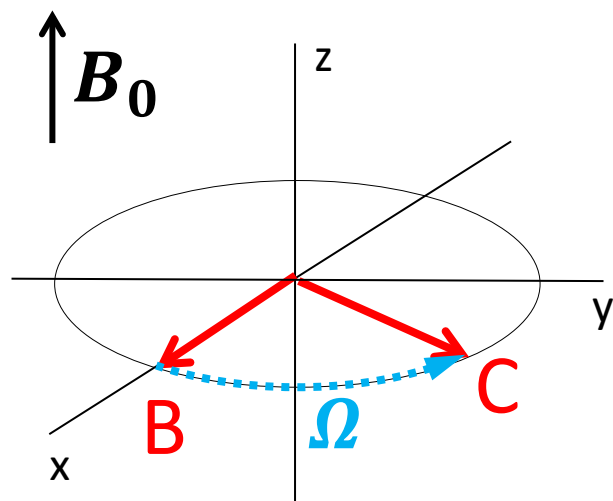
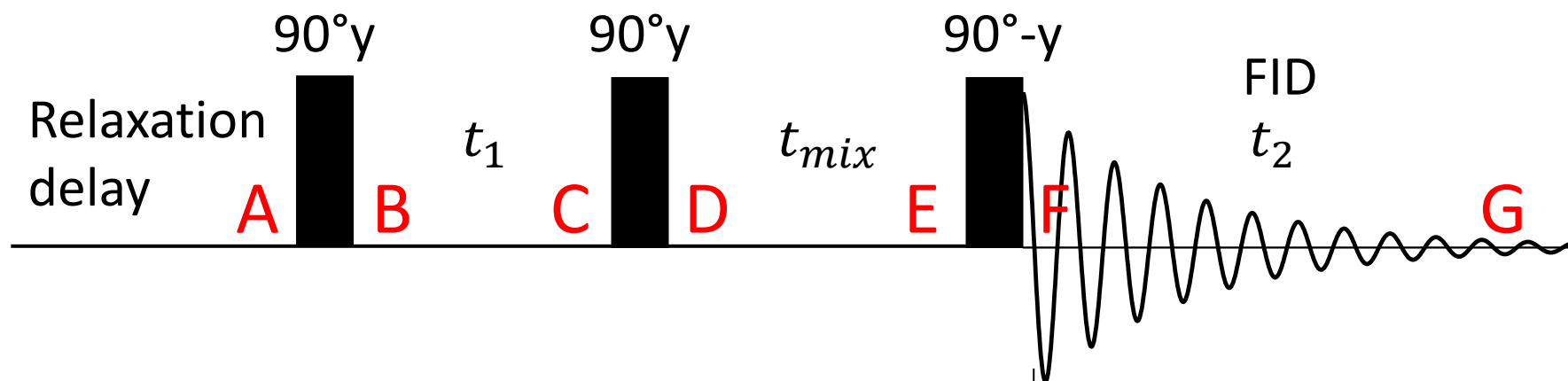
2D NMR spectroscopy



$$A: M_z = M_0$$

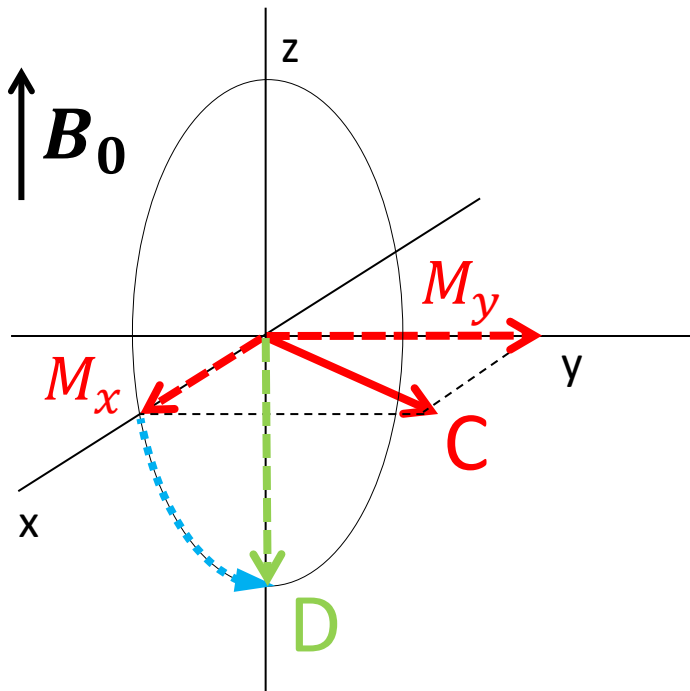
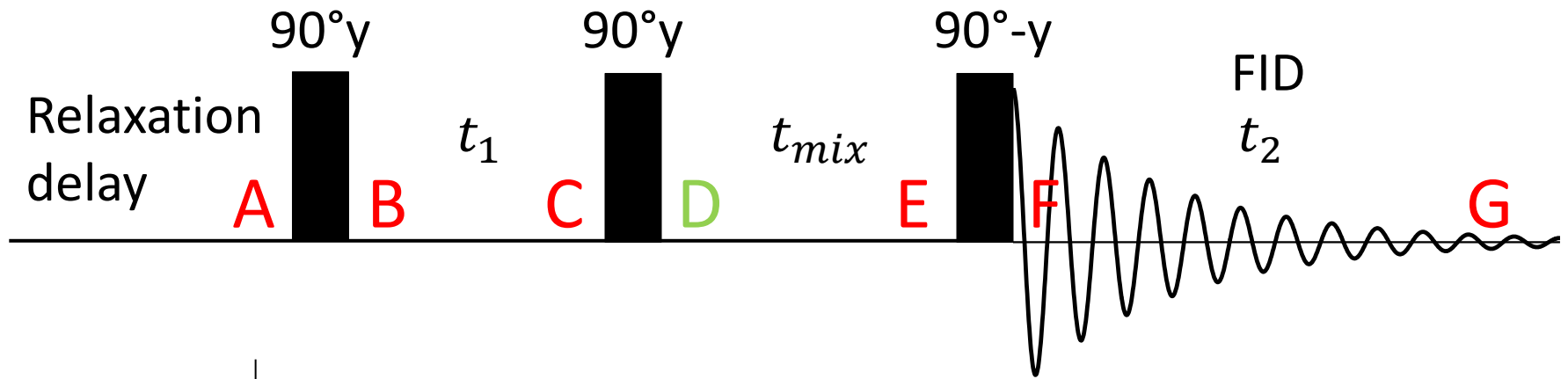
$$B: M_x = M_0$$

2D NMR spectroscopy



$$C: M_x = M_0 \cos \Omega t_1$$
$$M_y = M_0 \sin \Omega t_1$$

2D NMR spectroscopy



$$D: M_z = -M_0 \cos \Omega t_1$$

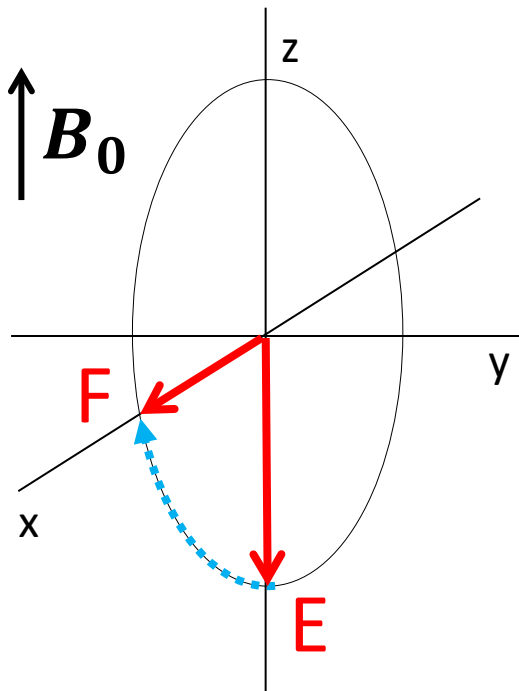
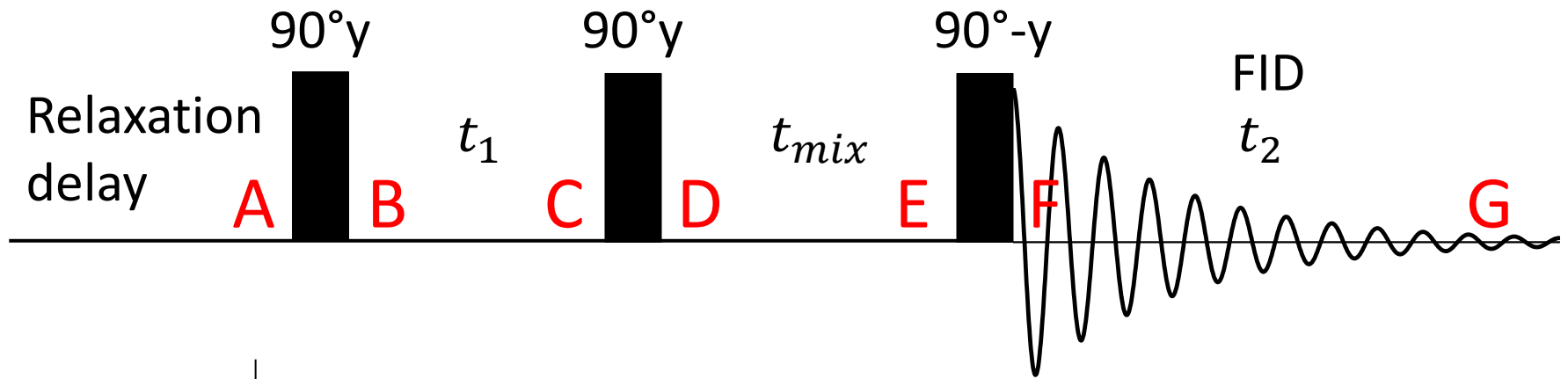
$$M_y = M_0 \sin \Omega t_1$$

(ignore M_y , it will get cancelled by phase cycling and/or gradients)

$$E: M_z = -M_0 \textcircled{a} \cos \Omega t_1$$

Relaxation during t_{mix}

2D NMR spectroscopy



$$E: M_z = -M_0 a \cos \Omega t_1$$

$$F: M_x = M_0 a \cos \Omega t_1$$

$$G: M_x = M_0 a \cos \Omega t_1 e^{-R_2 t_2} \cos \Omega t_2$$

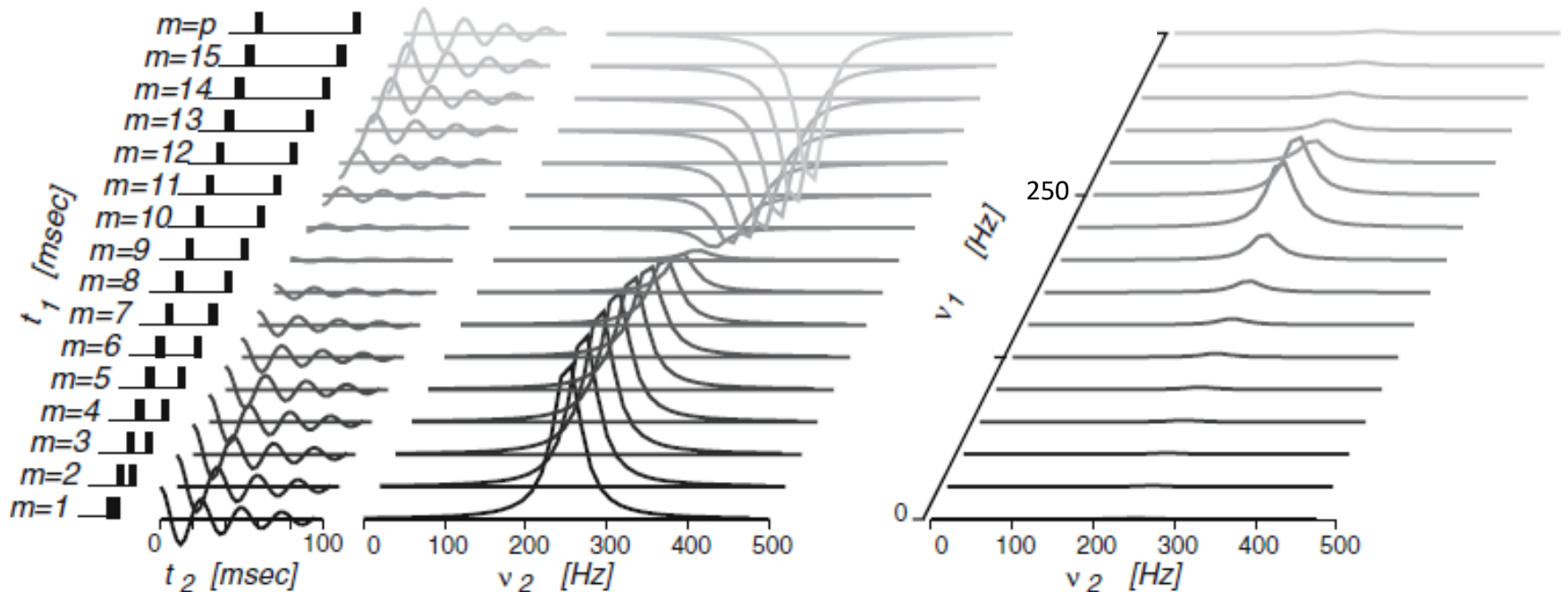
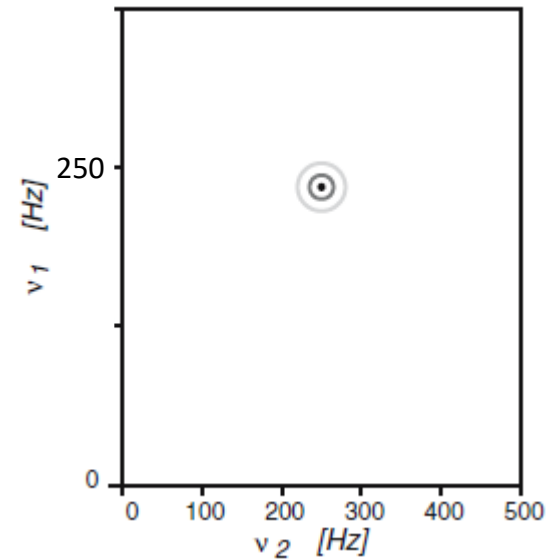
$$M_y = M_0 a \cos \Omega t_1 e^{-R_2 t_2} \sin \Omega t_2$$

$$M_{xy} = M_0 a \cos \Omega t_1 e^{-R_2 t_2} e^{i\Omega t_2}$$

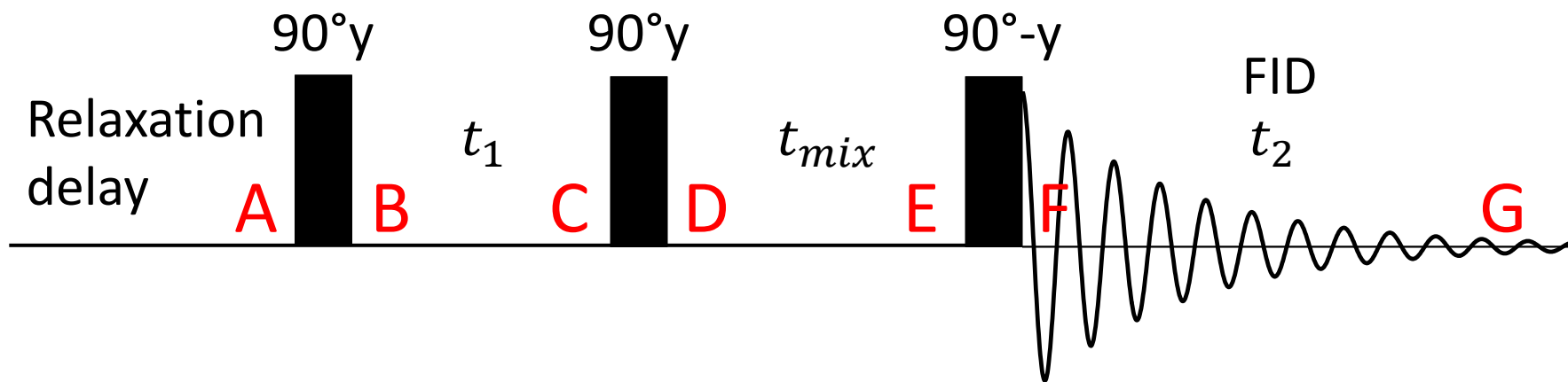
$$M_{xy} = M_0 a \boxed{\cos \Omega t_1} e^{-R_2 t_2} e^{i\Omega t_2}$$

Amplitude modulation

- Acquire data with increasing t_1
- Fourier transform each FID (in t_2)
- Fourier transform the spectra in t_1



2-dimensional (2D) NMR spectroscopy



Nucleus A

Nucleus B

$$D: M_z = -M_A \cos \Omega_A t_1$$

$$M_z = -M_B \cos \Omega_B t_1$$

$$E: -M_A a_{AA} \cos \Omega_A t_1$$

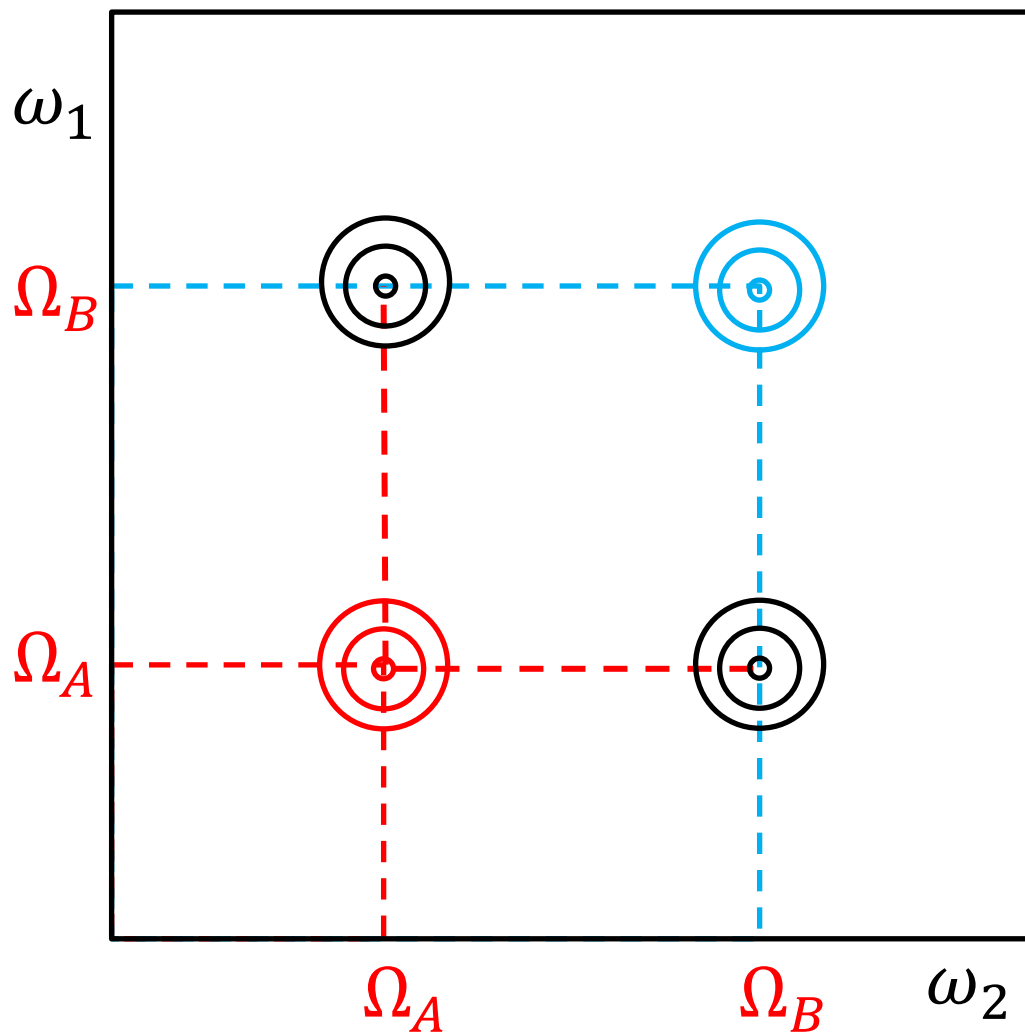
$$-M_B a_{BB} \cos \Omega_B t_1$$

$$-M_B a_{BA} \cos \Omega_B t_1$$

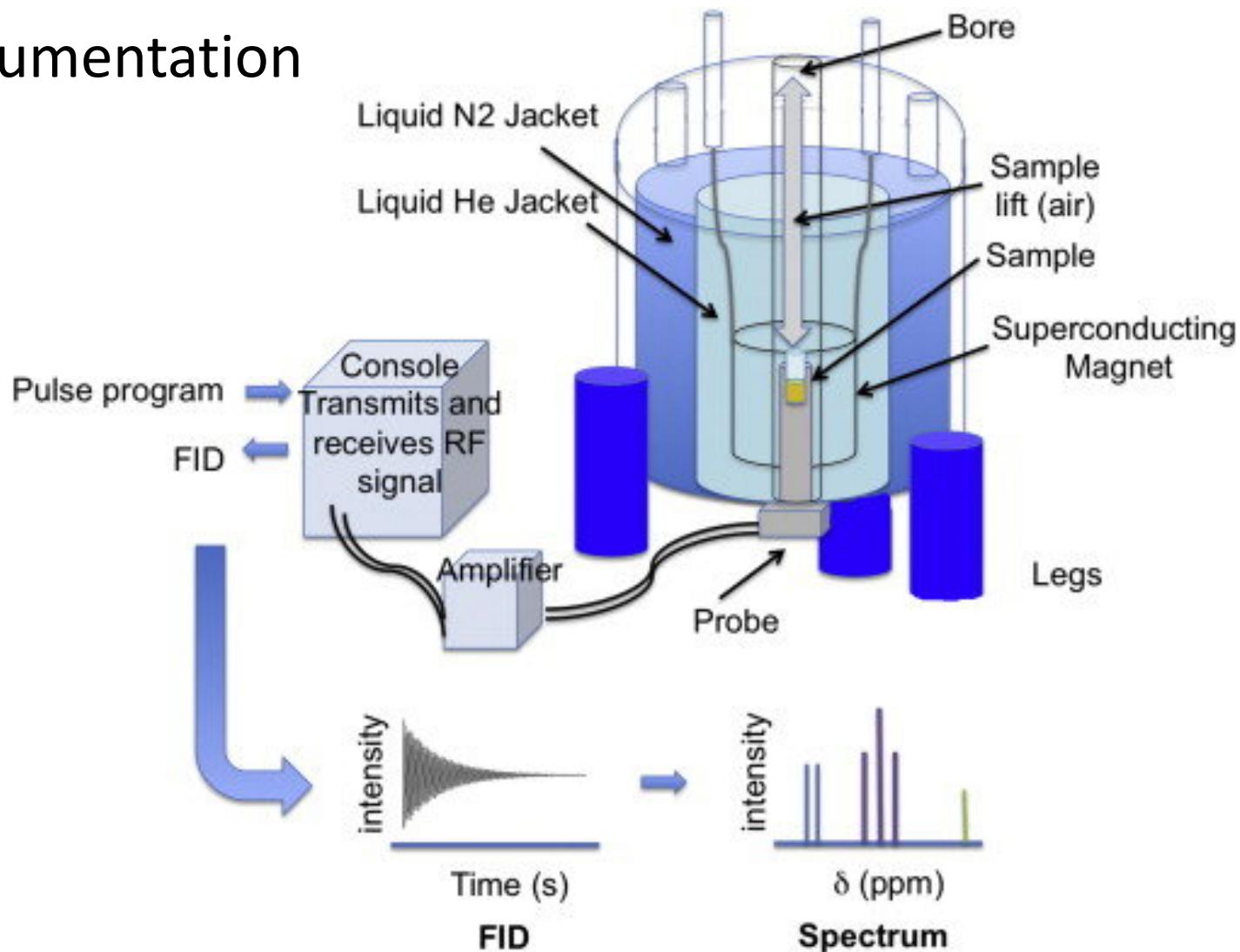
$$-M_A a_{AB} \cos \Omega_A t_1$$

$$G: () e^{-R_2 t_2} e^{i\Omega_A t_2}$$

$$() e^{-R_2 t_2} e^{i\Omega_B t_2}$$

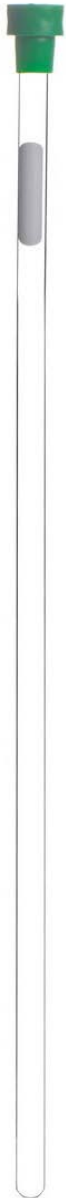


Instrumentation

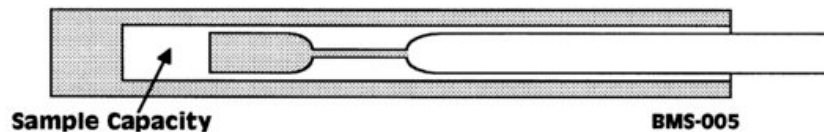


Rankin, Naomi & Preiss, David & Welsh, Paul & Burgess, Karl & Nelson, Scott & Lawlor, Debbie & Sattar, Naveed. (2014). The emergence of proton nuclear magnetic resonance metabolomics in the cardiovascular arena as viewed from a clinical perspective. *Atherosclerosis*. 237. 287–300.





- Typical NMR tube diameter: 5 mm
- Sample volume: > 600 μL
- Smaller volume (200-300 μL) in Shigemitsu tubes



- Concentration for structure determination: several hundred μM
- Lower concentration for ligand binding
- Signal to noise ratio (S/N) $\propto \sqrt{n}$
- Cryoprobe: electronics cooled to $\sim 20\text{K}$, higher S/N (up to 4 fold), lower concentrations possible

Setting up an NMR experiment

- Tuning: optimize efficiency of energy transfer between the coil and the sample
- ^2H Lock: long-term stability of the magnetic field
- Shimming: optimize homogeneity of the magnetic field within the sample
- Calibration of pulses: experimentally determine the duration of a 90° pulse for a given power level
- Data acquisition