Solutions to homework problem set 1:

Problem 1: a) Solve $y = x_1 + x_2X$ using Cramer's rule for the given dataset.

$$A = \begin{bmatrix} \sum_{i=1}^{n} \frac{1}{\sigma_i^2} & \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} \\ \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} & \sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \text{and } B = \begin{bmatrix} \sum_{i=1}^{n} \frac{D_i}{\sigma_i^2} \\ \sum_{i=1}^{n} \frac{D_i x_i}{\sigma_i^2} \end{bmatrix}$$

x	y	σ
2	10.6	1
4	12.1	1
6	14.5	1
8	20.8	1
10	17.3	1
12	24.7	1
14	29.1	1

Find the answer for x1 and x2 using Cramer's rule for the given data set and plot with a plotting program.
a)
a11 = 7*1 = 7
a12 = 2 + 4 + 6 + 8 + 10 + 12 + 14 = 56
a21 = a12 = 56
a22 = $2^2 + 4^2 + 6^2 + 8^2 + 10^2 + 12^2 + 14^2$ = 560
b1 = 10.6 + 12.1 + 14.5 + 20.8 + 17.3 + 24.7 + 29.1 = 129.1
b2 = 2*10.6 + 4*12.1 + 6*14.5 + 8*20.8 + 10*17.3 + 12*24.7 + 14*29.1 = 1199.8

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11} a_{22} - a_{12} a_{21}}, \quad x_2 = \frac{b_2 a_{11} - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

x1 = (129.1*560 – 1199.8*56)/(7*560 – 56*56) = 6.51428
x2 = (1199.8*7 – 7*560)/(7*560 – 56*56) = 1.49107 **[25]**
This problem could also be solved in different ways (matrix inversion, straight line simplified formulas from Internet), which is acceptable for full credit.

b) Extra credit: Write a C/C++ or Python program to solve this problem. **[10]**

(Check with Ayden for how to do it :)