

**Presenter: Emre Brookes**

**Topic:**

**Small angle X-ray and Neutron Scattering**

# *Outline - SAS*

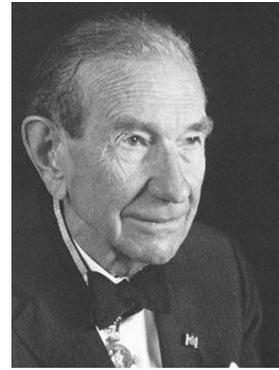
- Experimental setup
- Theory
- Modeling
- Software
- Practical considerations, Sample preparation etc.
- Possibly other techniques introduced as time permits

# Kratky plot

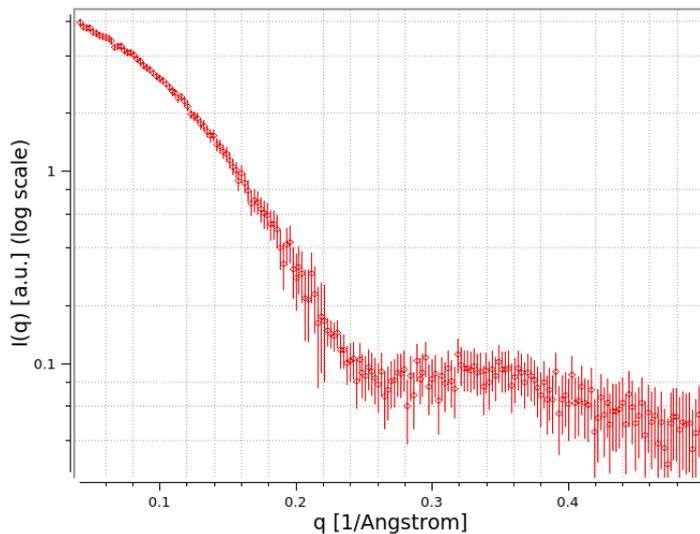
SAS provides a sensitive means to evaluate *the degree of compactness* of a protein:

- To determine whether a protein is globular, extended or unfolded
- To monitor the folding or unfolding transition of a protein

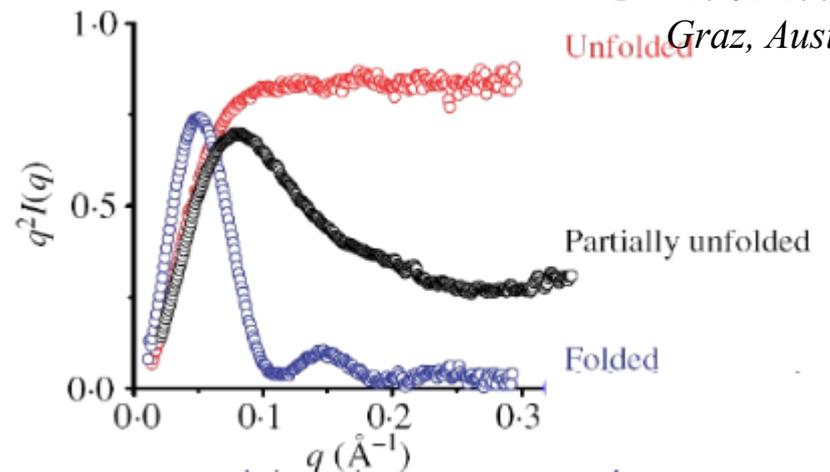
This is most conveniently represented using the Kratky plot:



Prof. Otto  
Kratky  
1902-1995  
Graz, Austria



$q^2 I(q)$  versus  $q$



Putnam, D., et al. (2007) *Quart. Rev. Biophys.* 40, 191-285.

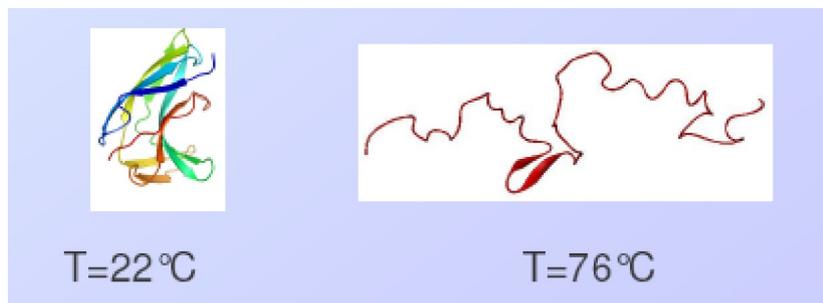
Folded particle : *bell-shaped curve* (asymptotic behavior  $I(q) \sim q^{-4}$ )

Random polymer chain : *plateau* at large  $q$ -values (asymptotic behavior  $I(q) \sim q^{-2}$ )

Extended polymer chain : *increase* at large  $q$ -values (asymptotic behavior  $I(q) \sim q^{-1.x}$ )

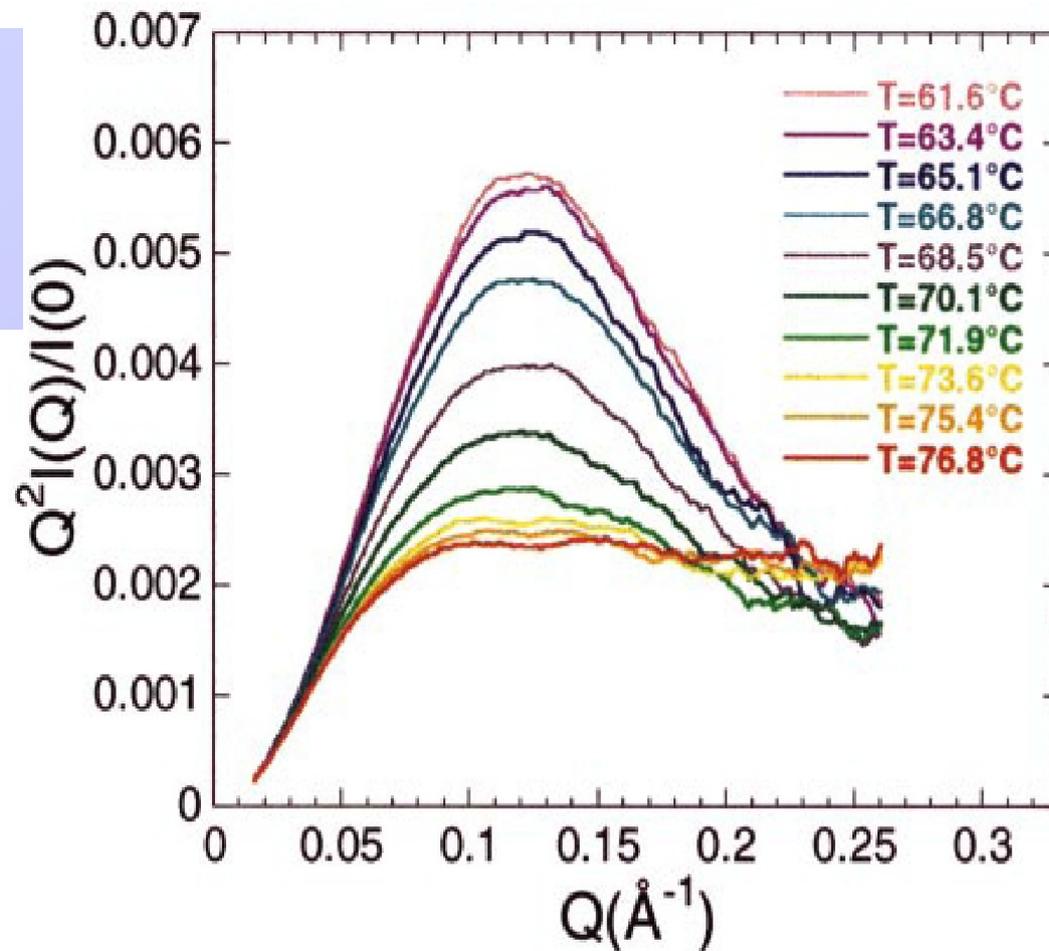
# Kratky plots of (partially) folded proteins

*Pérez et al., J. Mol. Biol.(2001), 308, 721-743*

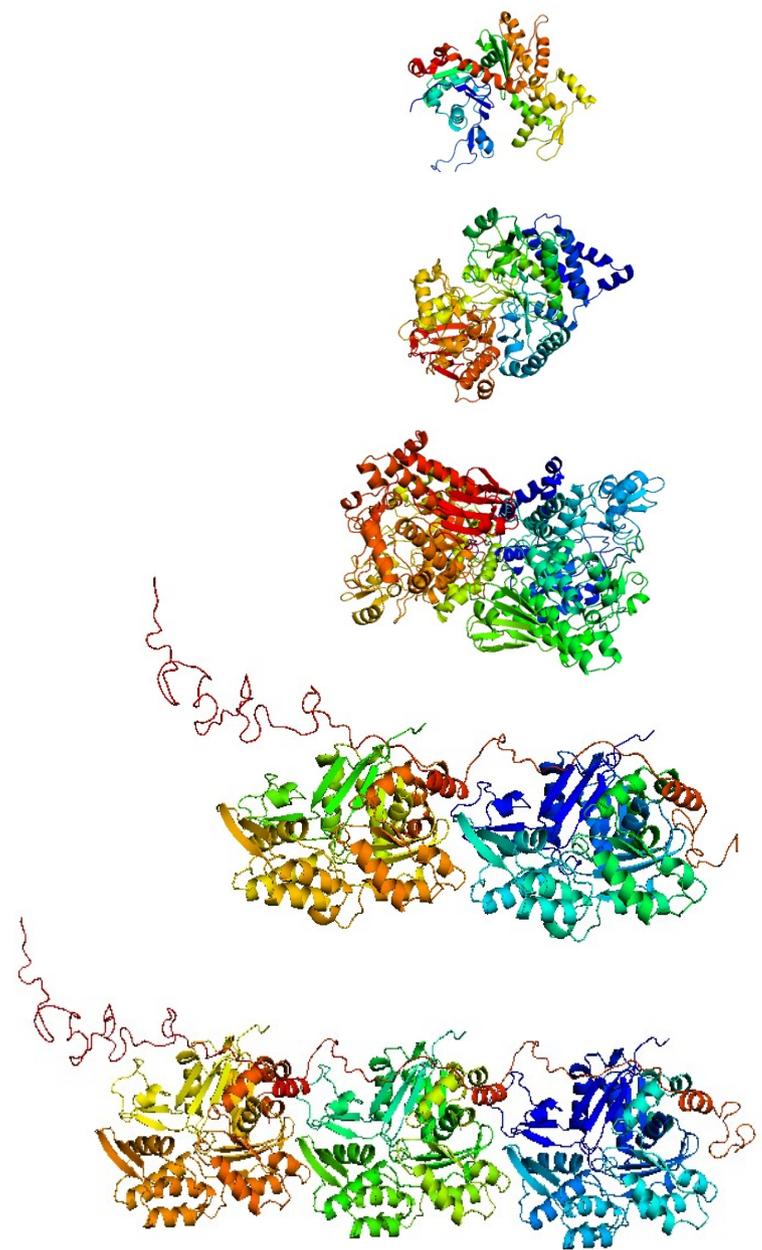
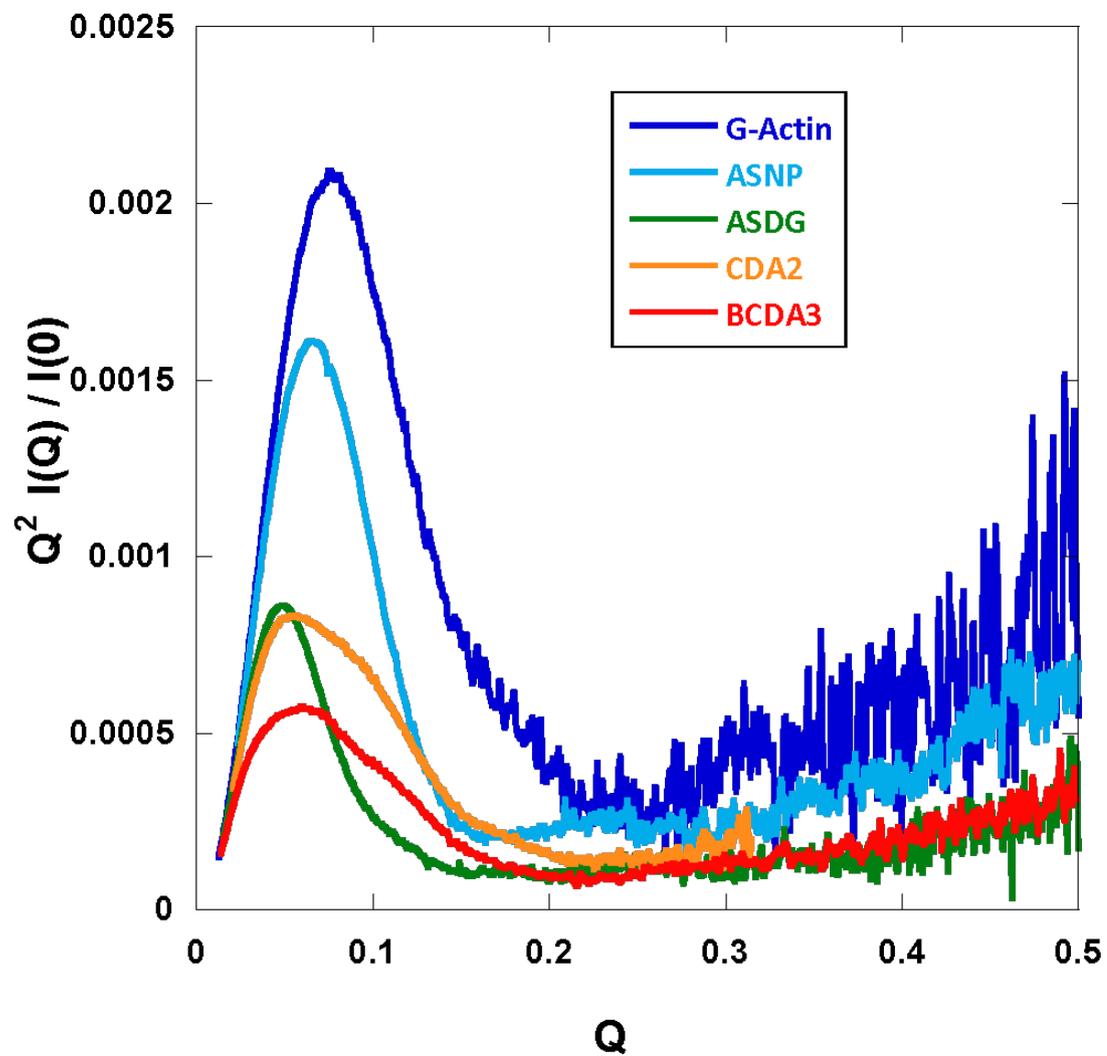


In practice, thin Gaussian chains do not exist.

In spite of the plateau at  $T=76^{\circ}\text{C}$ , NCS is not a Gaussian chain when unfolded, but a thick chain with persistence length.



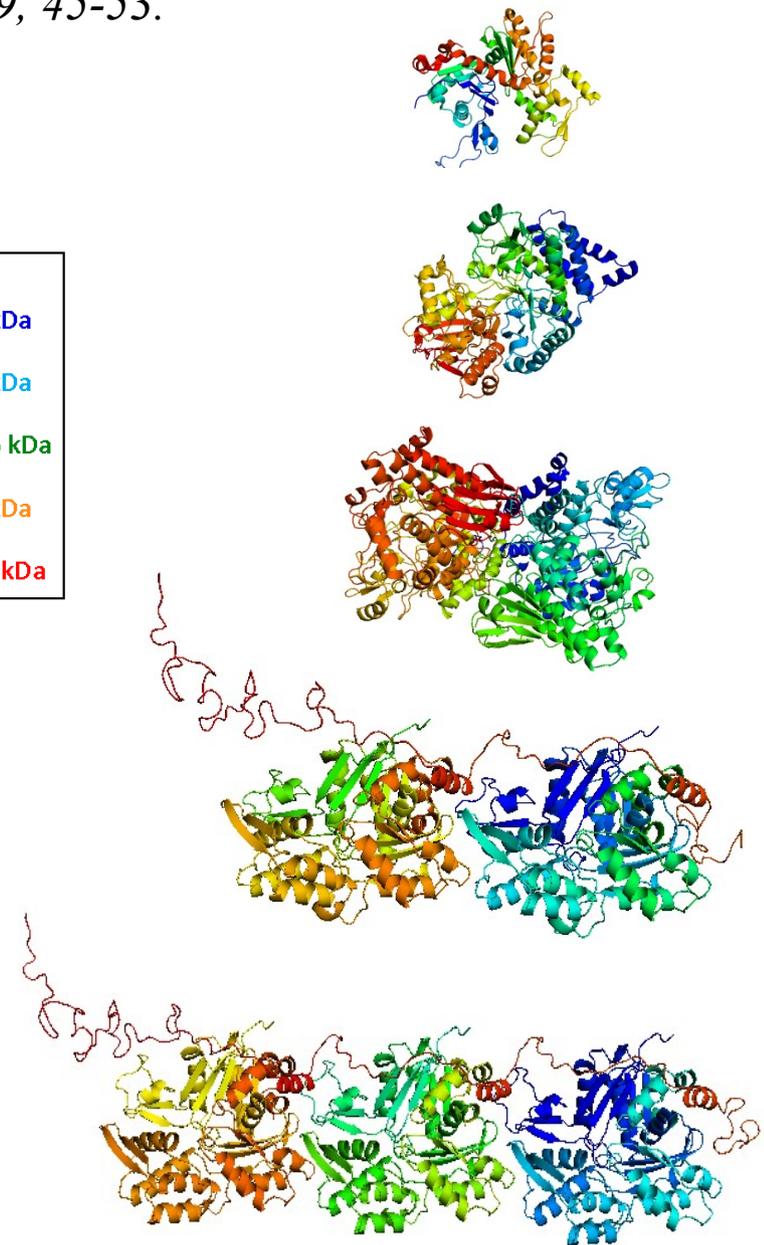
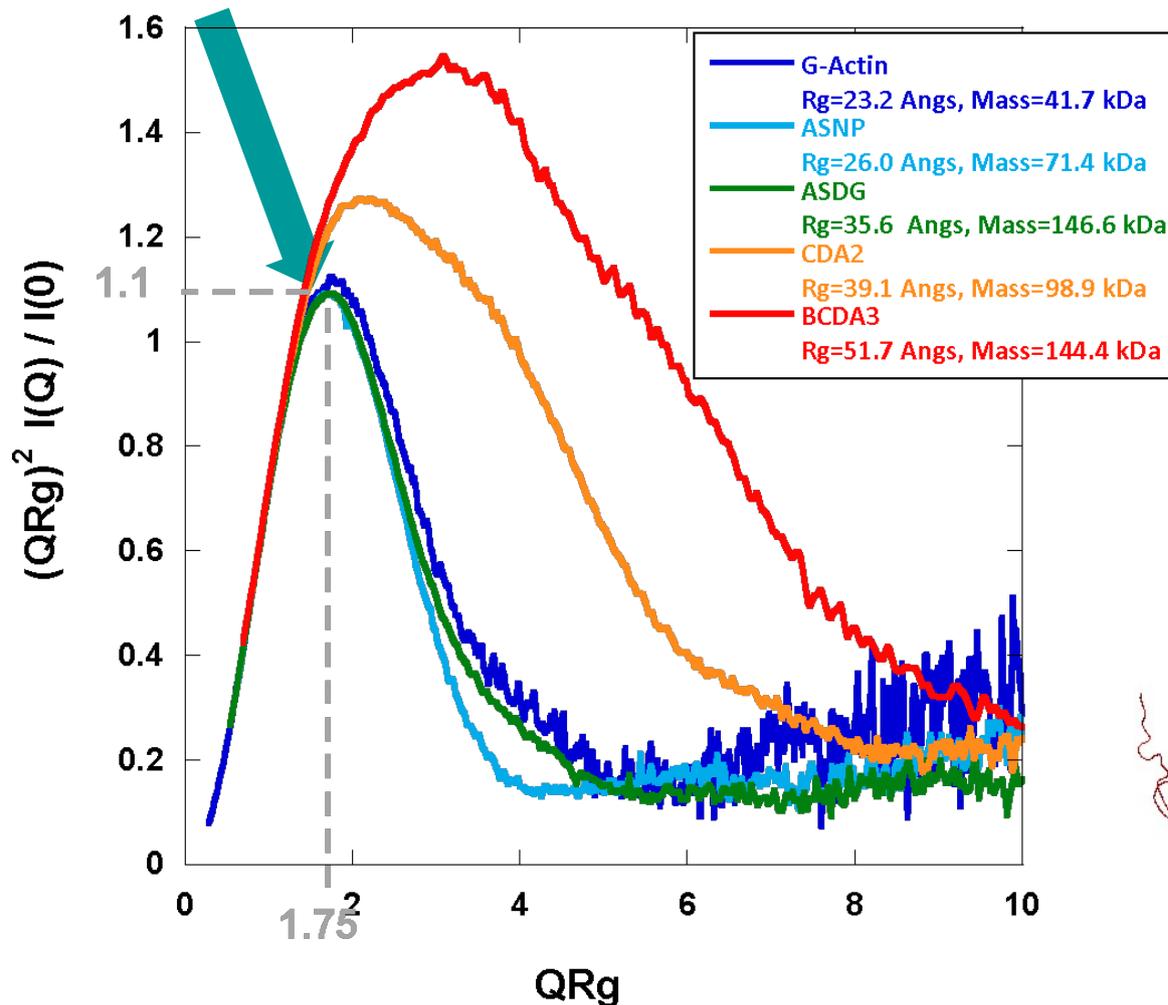
# Kratky plots of folded proteins



# Dimensionless Kratky plots of folded proteins

*Durand et al. (2010), J. Struct. Biol. 169, 45-53.*

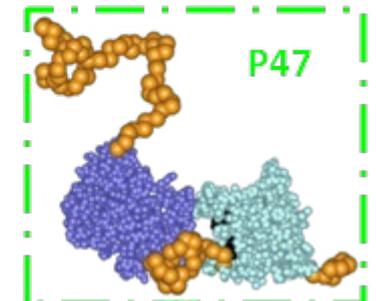
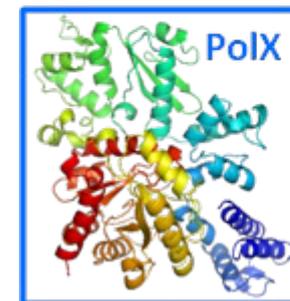
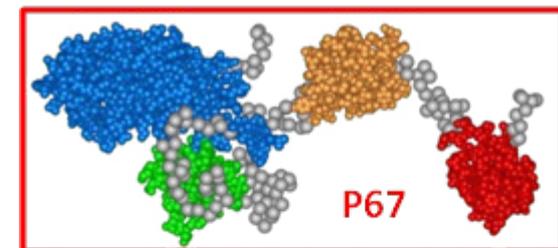
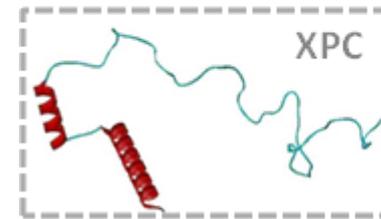
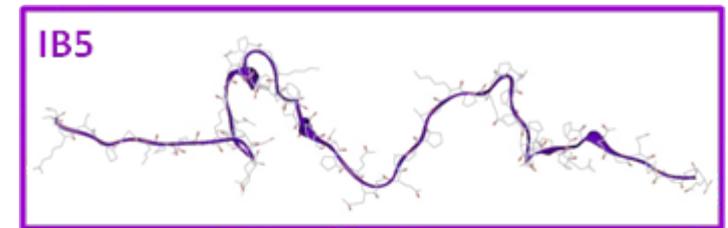
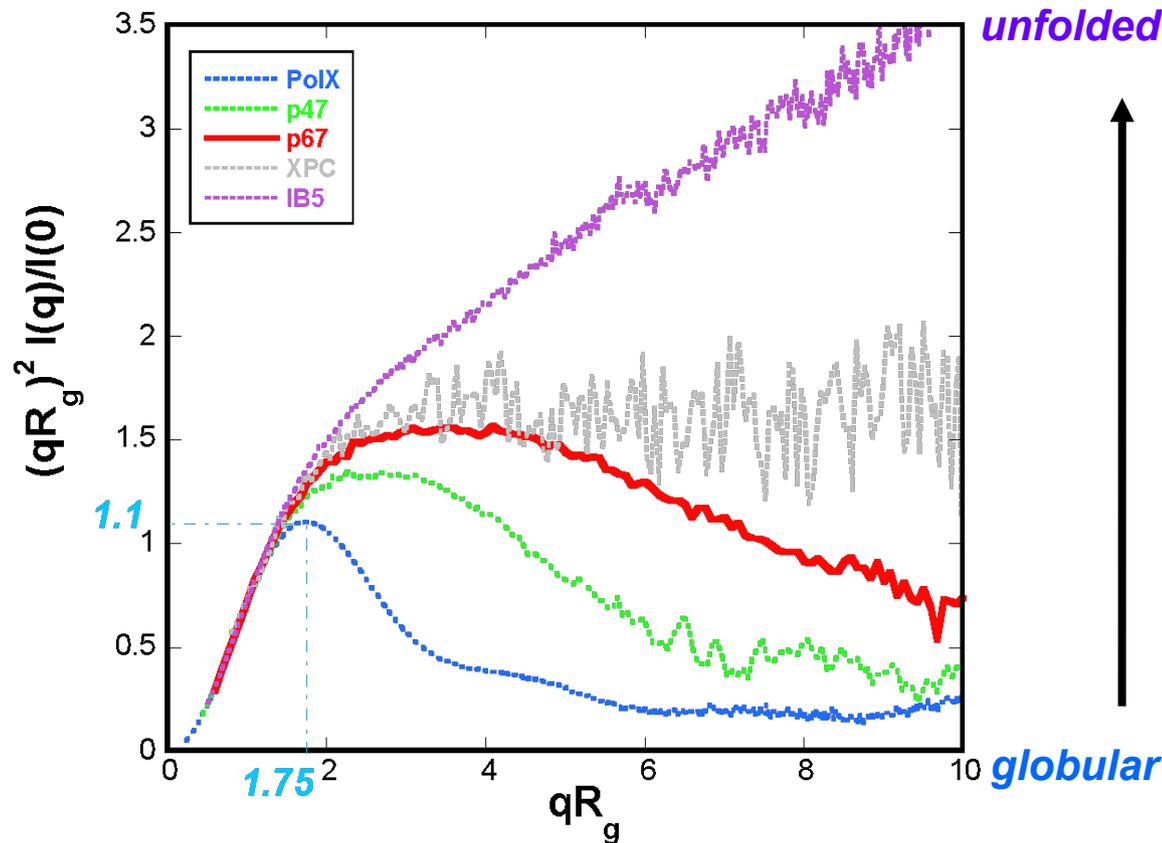
For globular structures, DLKPs fold into the same maximum



The maximum value on the dimensionless bell shape tells if the protein is globular.

# Dimensionless Kratky plots of (partially) folded proteins

Receveur-Bréchet V. and Durand D (2012), *Curr. Protein Pept. Sci.*, 13:55-75.

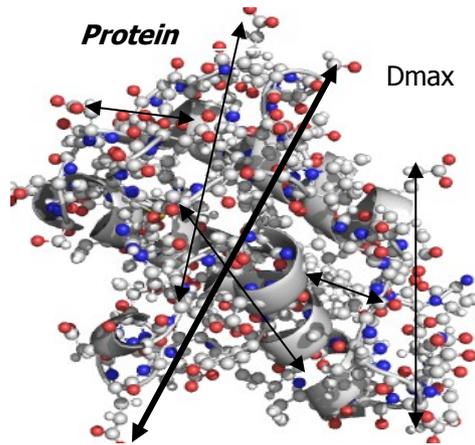


The bell shape vanishes as folded domains disappear and flexibility increases.

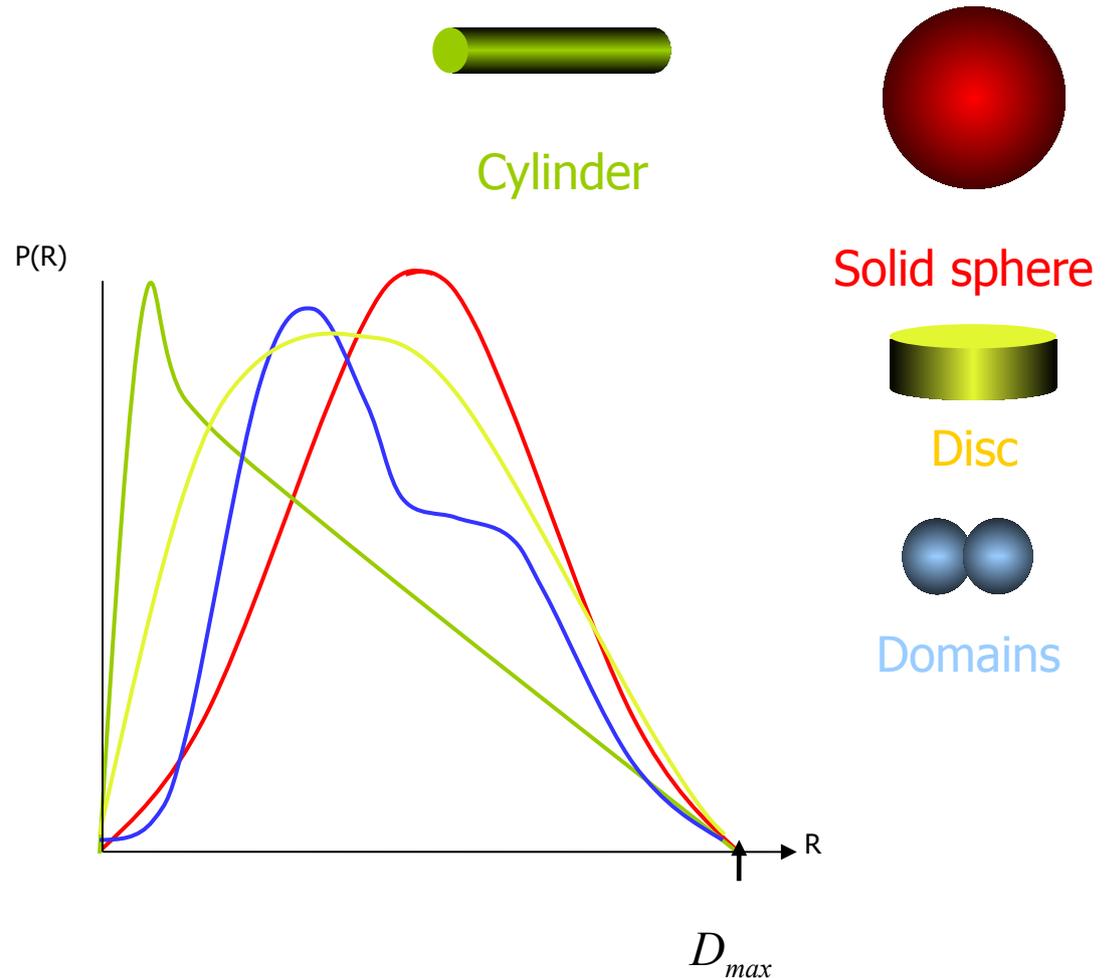
The curve increases at large  $q$  as the structure extends.

# Distance distribution function $p(r)$

$p(r)$  is obtained by “histogramming” the distances between any pair of scattering elements within the particle. (weighted by scattering density)



$p(r)$  vanishes at  $r = D_{max}$



The distance distribution function characterizes the shape of the particle **in real space**

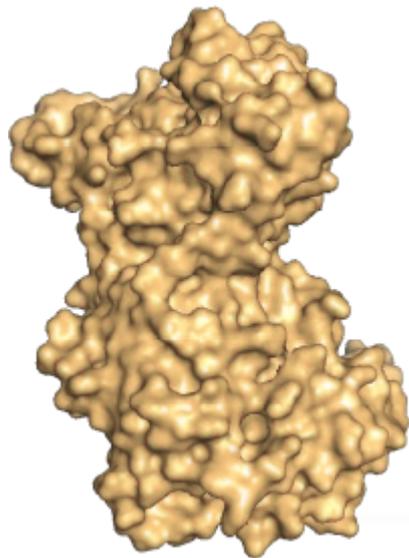
# Autocorrelation function

$$\gamma(\mathbf{r}) = \rho(\mathbf{r}) * \rho(-\mathbf{r}) = \int_{V_{\mathbf{u}}} \rho(\mathbf{r} + \mathbf{u}) \rho(\mathbf{u}) dV_{\mathbf{u}}$$

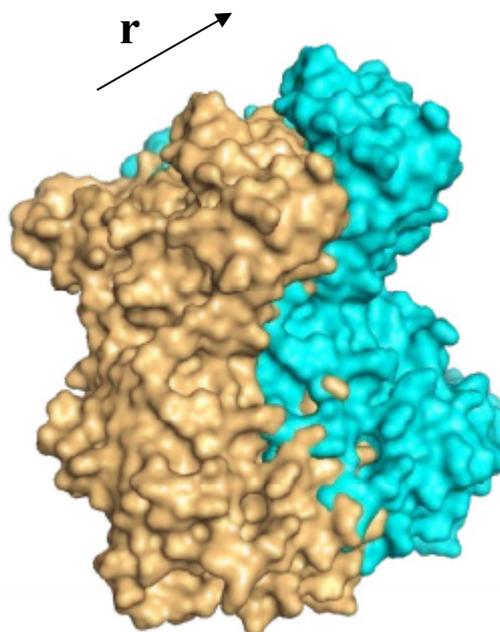
$\rho(\mathbf{r}) = \rho$       Assuming uniform scattering density inside of "Molecule", 0 o.w.

$$\Rightarrow \gamma(\mathbf{r}) = \rho^2 V_{\text{overlap}}(\mathbf{r}) \quad \gamma(0) = \rho^2 V$$

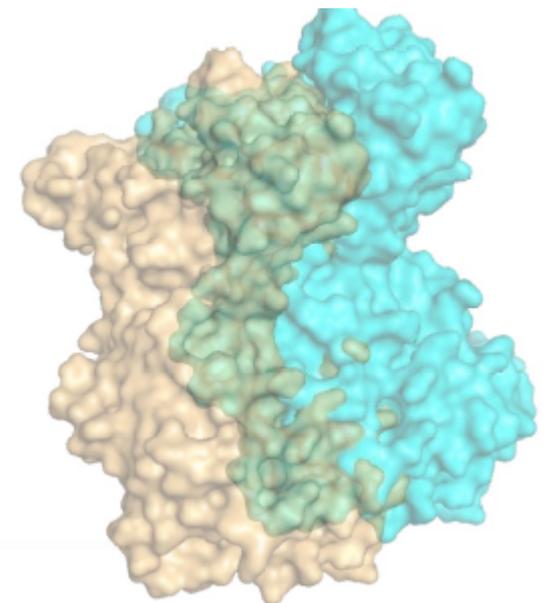
Molecule



Ghost



Molecule  $\cap$  Ghost = overlap



*Images: Patrice Vachette*

# Autocorrelation function

$$\gamma(\mathbf{r}) = \rho(\mathbf{r}) * \rho(-\mathbf{r}) = \int_{V_{\mathbf{u}}} \rho(\mathbf{r} + \mathbf{u}) \rho(\mathbf{u}) dV_{\mathbf{u}}$$

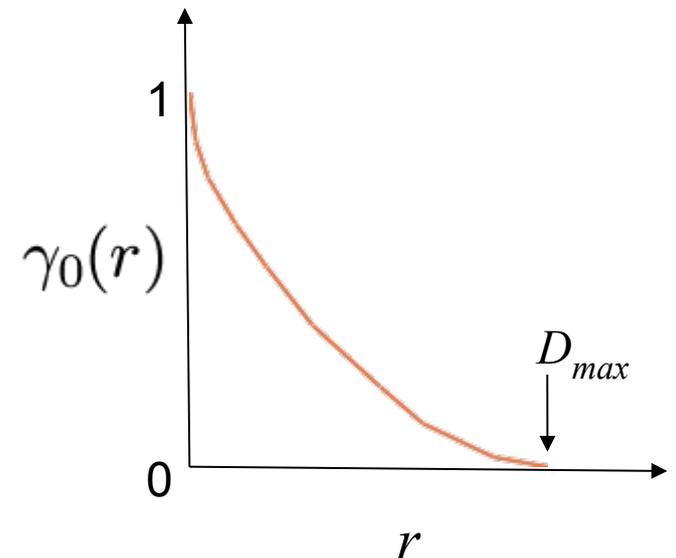
$\rho(\mathbf{r}) = \rho$  Assuming uniform scattering density inside of "Molecule", 0 o.w.

$$\Rightarrow \gamma(\mathbf{r}) = \rho^2 V_{\text{overlap}}(\mathbf{r}) \quad \gamma(0) = \rho^2 V$$

$\gamma(r) = \langle \gamma(\mathbf{r}) \rangle$  Spherical average

$\gamma_0(r) = \frac{\gamma(r)}{\gamma(0)}$  Characteristic function

$\gamma_0(r)$  Probability of finding a point within the molecule at a distance  $r$  from a given point

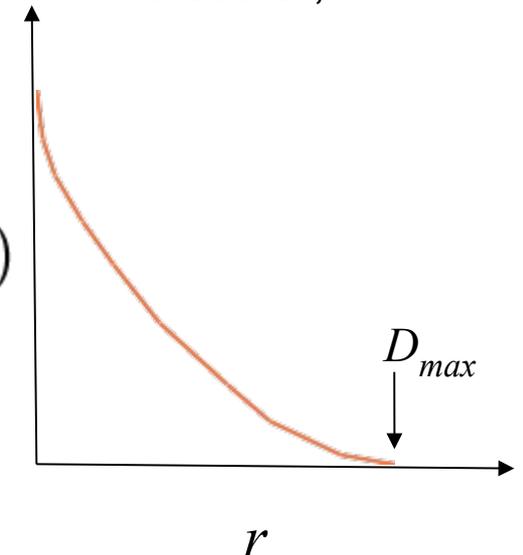
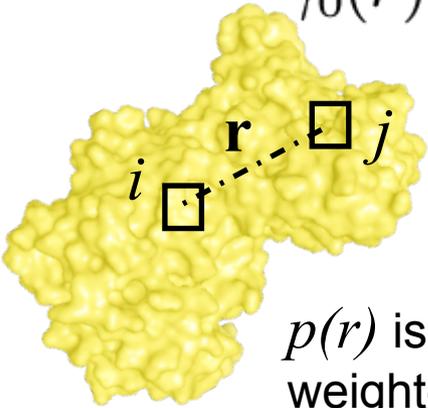


# Distance (pairwise) distribution function

$\rho(\mathbf{r}) = \rho$  Assuming uniform scattering density inside of “Molecule”, 0 o.w.

$$\Rightarrow \gamma(\mathbf{r}) = \rho^2 V_{\text{overlap}}(\mathbf{r}) \quad \gamma(0) = \rho^2 V$$

$\gamma_0(r)$  Probability of finding within the molecule a point  $j$  at a distance  $r$  from a given point  $i$



$p(r)$  is the distribution of distances between all pairs of points within the molecule weighted by the respective scattering densities.

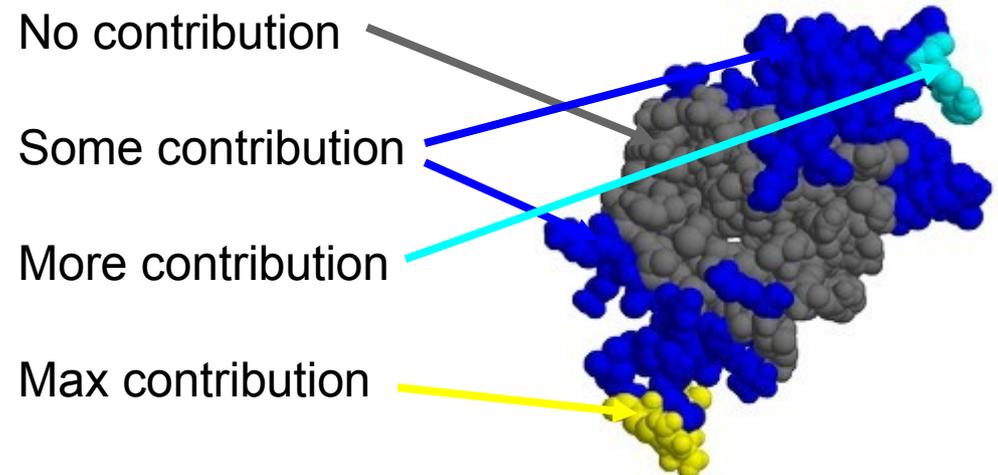
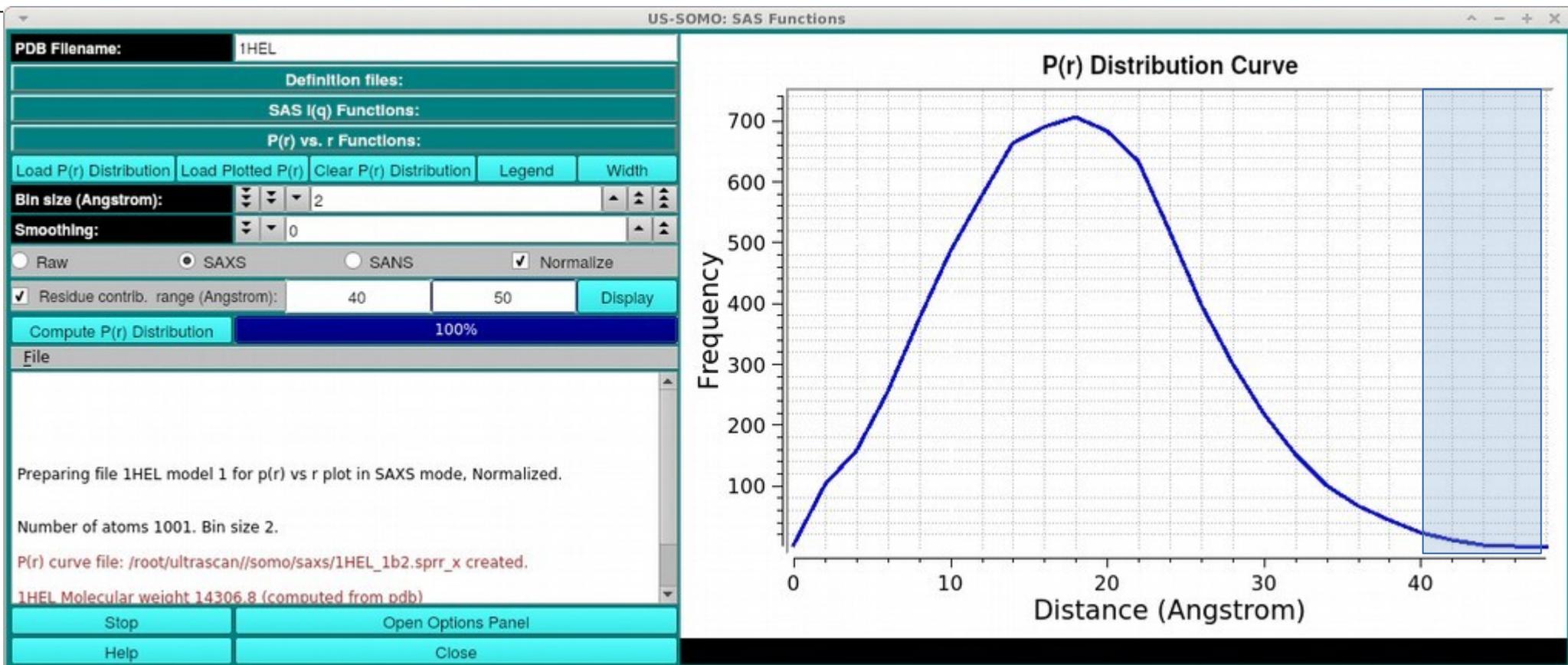
→ number of elementary volumes  $i \propto V$

→ number of elementary volumes  $j \propto r^2$

→ number of pairs  $(i,j)$  separated by the distance  $r \propto V r^2 \gamma_0(r)$

$$p(r) = \rho^2 V r^2 \gamma_0(r) = r^2 \gamma(r)$$

# Distance distribution function $p(r)$



## ***Relation between $p(r)$ and $I(q)$***

$$I(q) = 2\pi^2 \int_0^\infty p(r) \frac{\sin(qr)}{qr} dr$$

$$p(r) = \frac{1}{2\pi^2} \int_0^\infty I(q) qr \sin(qr) dq$$

However, direct calculation of  $p(r)$  from  $I(q)$  is made difficult and risky by  $[q_{min}, q_{max}]$  truncation and data noise effects.

# Back-calculation of the Distance Distribution Function

Glatter, O. *J. Appl. Cryst.* (1977) **10**, 415-421.

Main hypothesis : the particle has a finite size, characterised by  $D_{max}$ .

$D_{max}$  is proposed by the user

$p(r)$  is described over  $[0, D_{Max}]$  by a linear combination of  $M$  orthogonal functions

$$p_{\text{theoret}}(r) = \sum_{n=1}^M c_n \varphi_n(r)$$

$I(q)$  is calculated by Fourier Transform of  $p_{\text{theoret}}(r)$

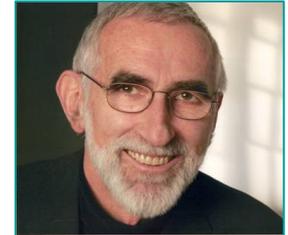
$$I(q) = 2\pi^2 \int_0^{D_{\max}} p_{\text{theoret}}(r) \frac{\sin(qr)}{qr} dr$$

Svergun (1988) : program "GNOM"

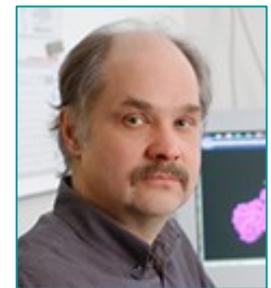
$M \sim 30 - 100$ ; ill-posed least squares regularization method

“Perceptual criteria” : smoothness, stability, absence of systematic deviations

- Each criterium has a predefined weight
- The solution is given a score calculated by comparison with “ideal values”



Prof. Otto Glatter  
Guinier Prize 2012  
Graz, Austria



Dr. Dmitri Svergun  
Hamburg, Germany

# Back-calculation of the Distance Distribution Function

$$p(r) = \frac{r^2}{2\pi^2 r_e^2 \varphi} \int_0^\infty q^2 I(q) \frac{\sin(qr)}{qr} dq$$

$$p_{\text{theoret}}(r) = \sum_{n=1}^M c_n \varphi_n(r)$$

The method of Glatter requires prior estimates of  $D_{max}$  and a regularization parameter. Steen introduced methods to determine these parameters automatically using Bayesian analysis methods.



*Thomas Bayes*  
1701-1761

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

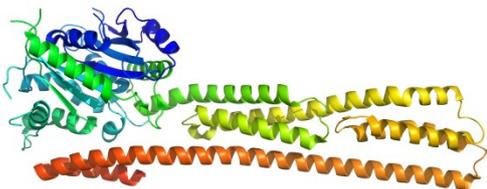
Data –  $I(q)$   
Generative model (Glatter)  
Priors –  $D_{max}$ ,  $\alpha$

Bayesian inference:

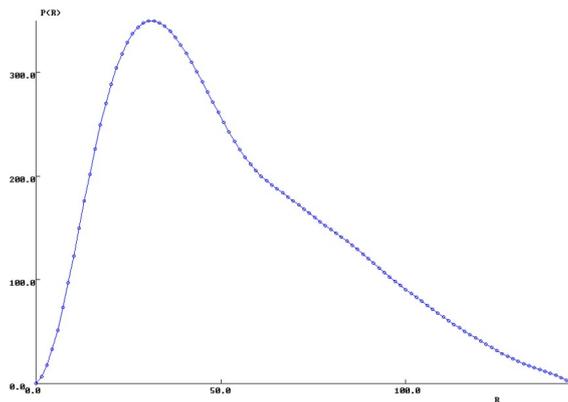
Prior dists → sample → generate → filter against data → Posterior dists

# Distance Distribution Function

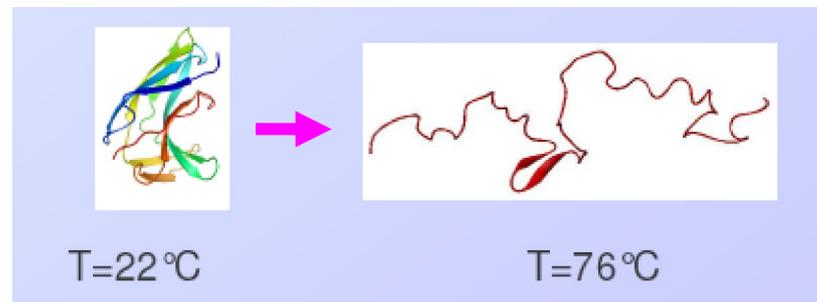
GBP1



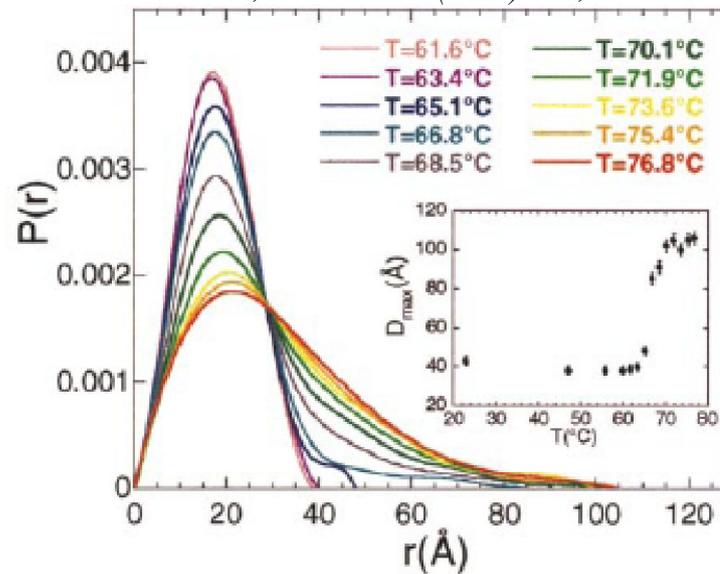
Real space:  $R_g = 42.34$  ,  $I(O) = 0.2775E+06$



Heat denaturation of Neocarzinostatin



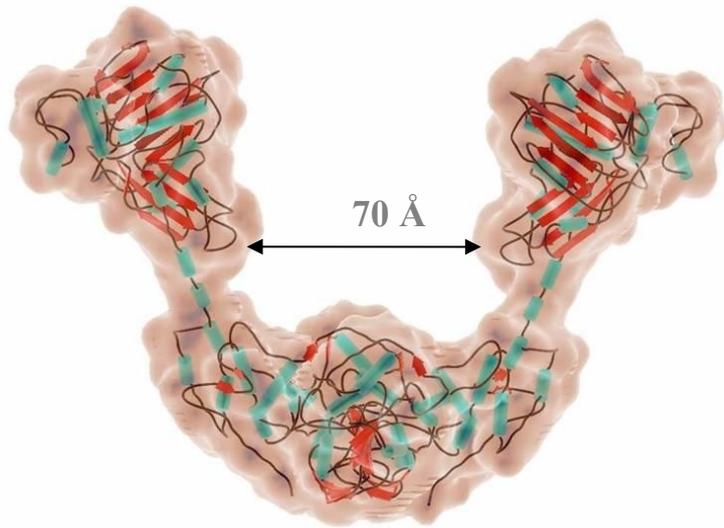
*Pérez et al., J. Mol. Biol. (2001) 308, 721-743*



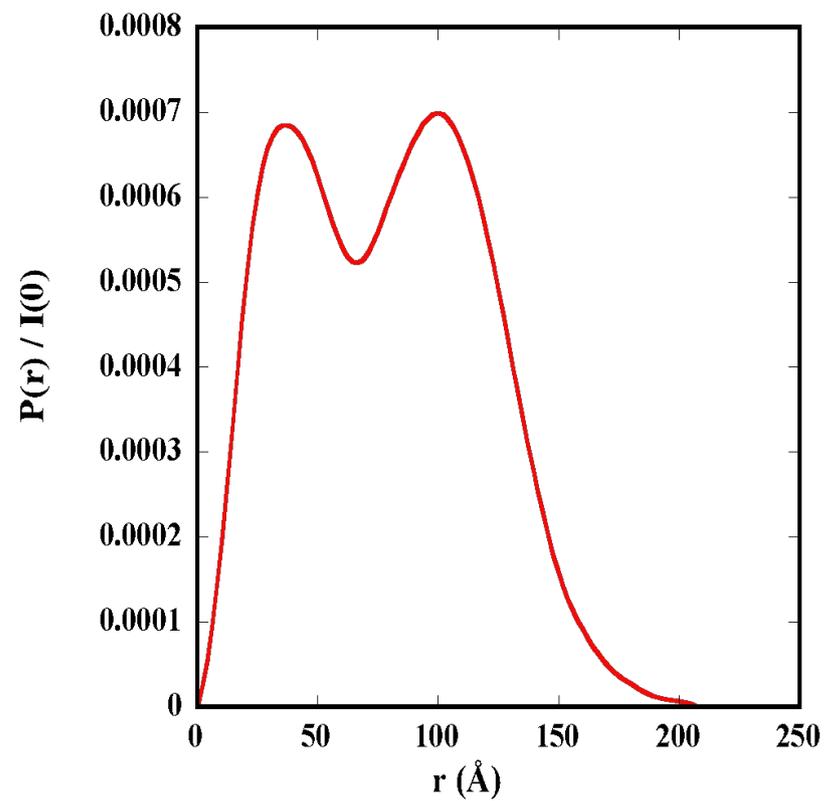
# Distance Distribution Function

*M. Graille et al., Structure (2008), 16, 360-370.*

*Topoisomerase VI*



Bimodal distribution



# Distance Distribution Function

The radius of gyration and the intensity at the origin can be derived from  $p(r)$  using the following expressions :

$$R_g^2 = \frac{\int_0^{D_{\max}} r^2 p(r) dr}{2 \int_0^{D_{\max}} p(r) dr}$$

$$I(0) = 4\pi r_e^2 \varphi \int_0^{D_{\max}} p(r) dr$$

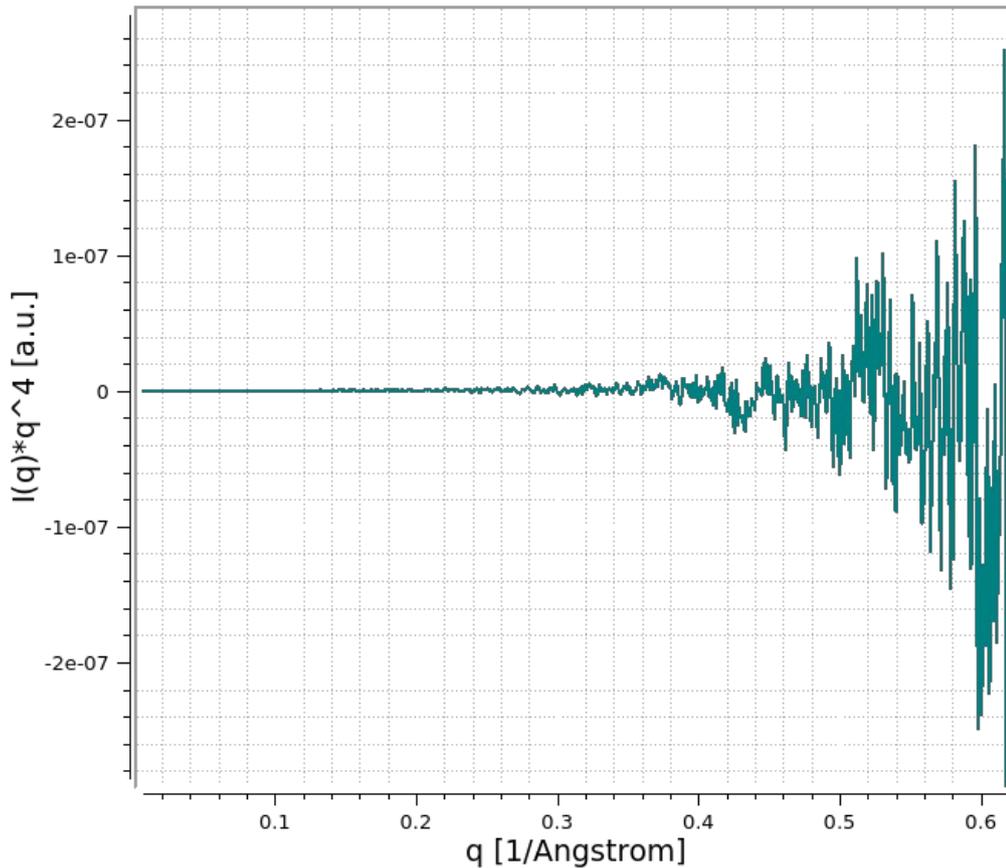
This alternative estimate of  $R_g$  makes use of the whole scattering curve, and is less sensitive to interactions or to the presence of a small fraction of oligomers.

Comparison of estimates from Guinier analysis and from  $P(r)$  is a useful cross-check.

# Porod's law

$$I \sim q^{-4}$$

Intensity decay is proportional to  $q^{-4}$  at higher angles for globular particles of uniform density.



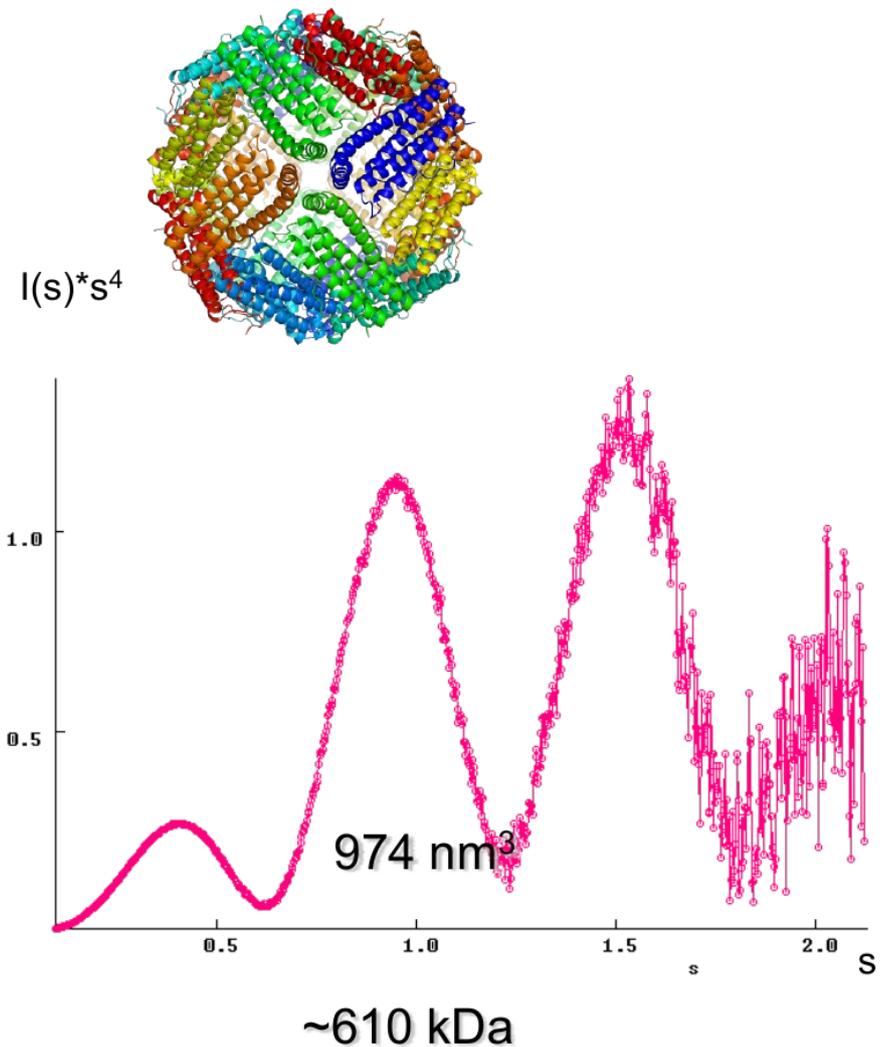
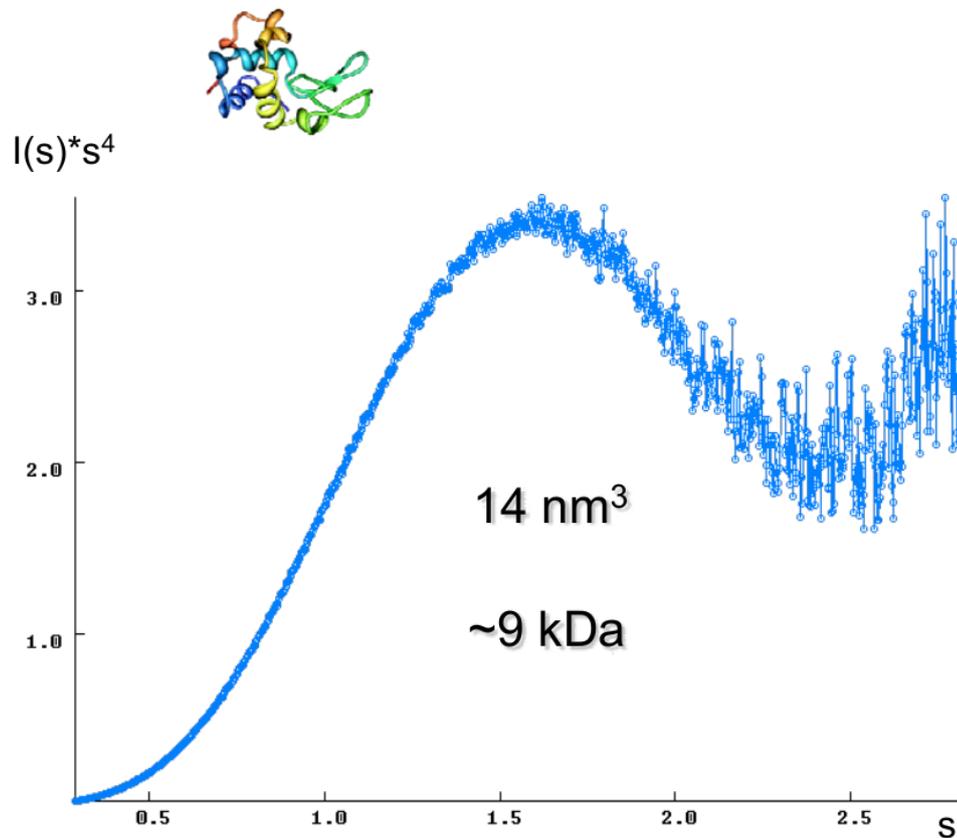
*Günther  
Porod  
1919-1984  
Graz, Austria*

$$V_P = \frac{2\pi^2 I(0)}{\int_0^\infty [I(q) - K_4] q^2 dq}$$

# Porod's law

$$V_P = \frac{2\pi^2 I(0)}{\int_0^\infty [I(q) - K_4] q^2 dq}$$

$K_4$  is a constant determined to ensure the asymptotic decay of  $I(q)$  is proportional to  $q^{-4}$

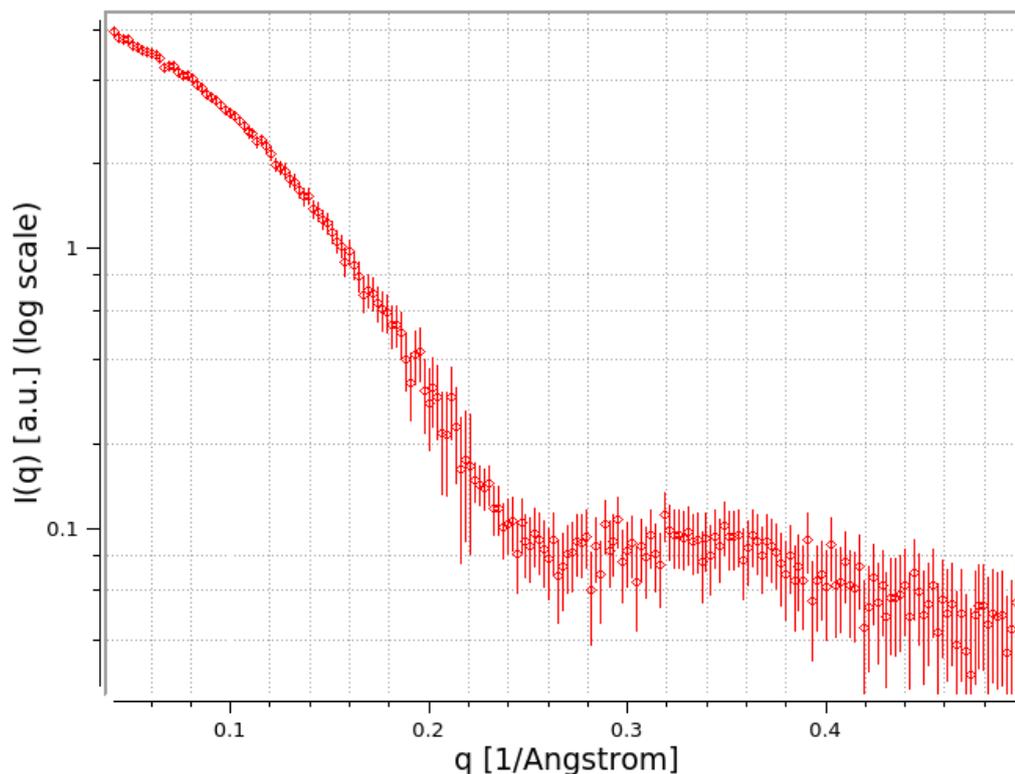


*Images courtesy Al Kikhney, EMBL*

# Information in a SAS curve

*Svergun, D.I. & Koch, M.H.J. (2003) Small-angle scattering studies of biological macromolecules in solution. Rep. Prog. Phys. 66 1735-82*

- Shannon channels =  $D_{max} \cdot q\text{-range} / \pi$
- “the number of [obtainable parameters] typically does not exceed **10–15**”



Lysozyme  $D_{max} \sim 48$  Angstroms

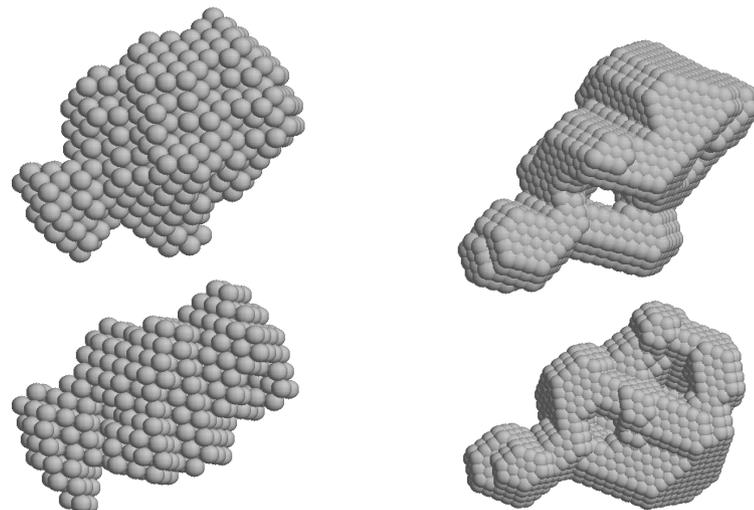
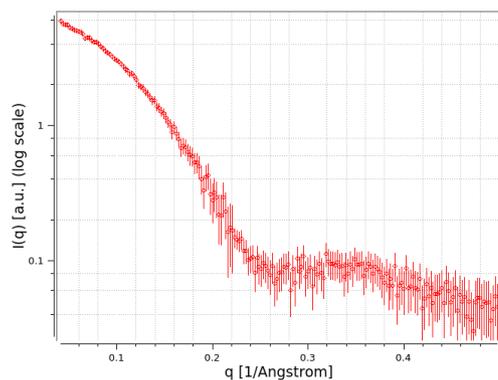
Shannon channels =  $48 * 0.5 / \pi \sim 8$

# Information in a SAS curve

*V. V. Volkov and D. I. Svergun (2003). Uniqueness of ab-initio shape determination in small-angle scattering. J. Appl. Cryst. 36, 860-864.*

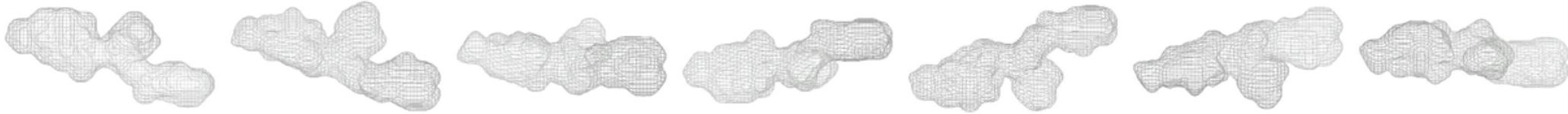
DAMAVER is a set of programs to align ab initio low resolution models (e.g. provided by DAMMIN, DAMMIF and/or GASBOR), select the most typical ("probable") one and build an averaged model.

- DAMSEL: compare all models, find most probable one and outliers
- DAMSUP: align all models with the most probable one
- DAMAVER: average aligned models and compute probability map
- DAMFILT: filter the averaged model at a given cut-off volume
- DAMSTART: generates from the averaged model an input file with fixed core for DAMMIN (for those who want to refine the averaged model)

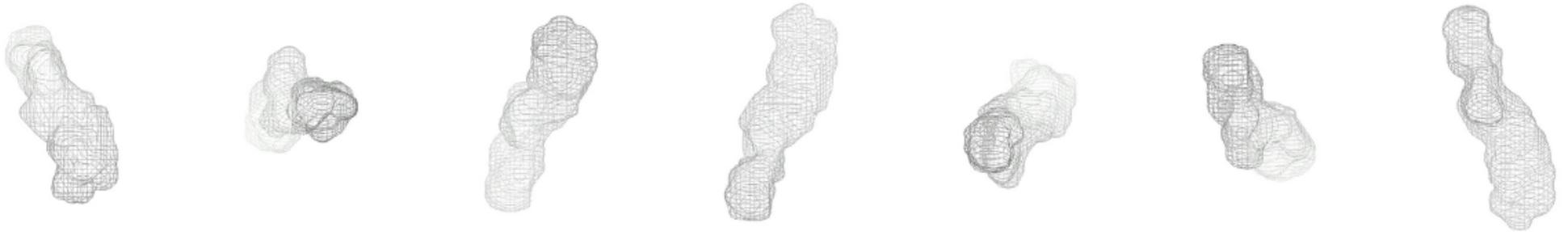


# Ab initio models

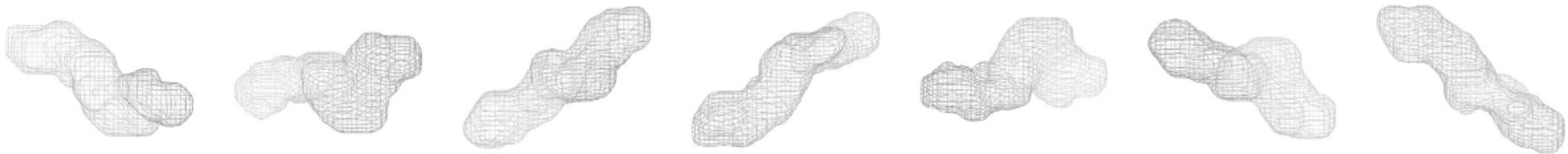
TGF $\beta$  / APS ID-12 / DAMMIN



est-2



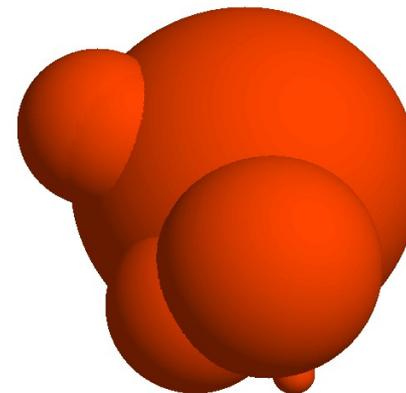
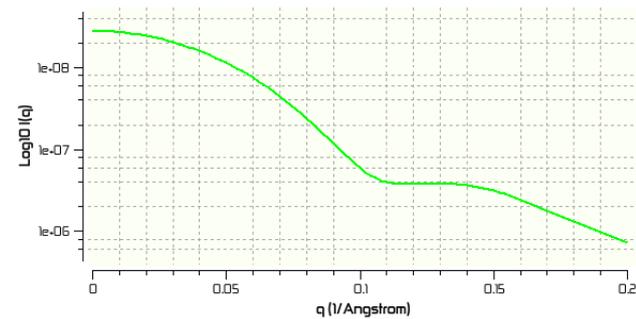
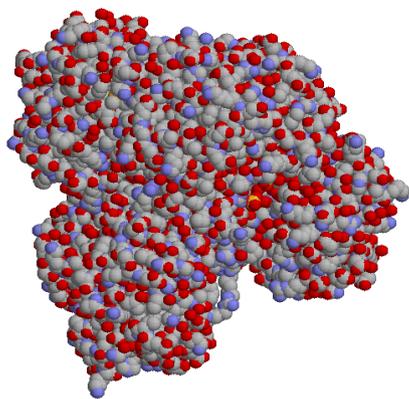
est-3



est-00



# Parsimonious modeling

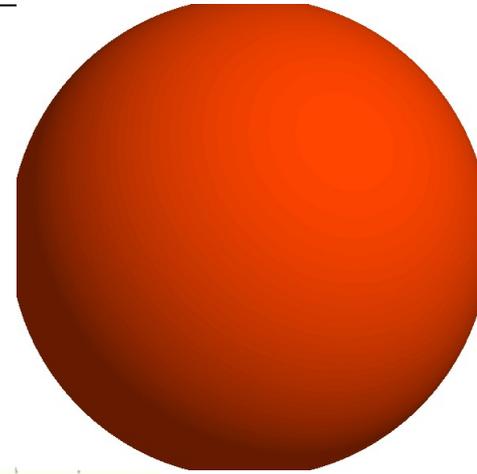


# Parsimonious modeling

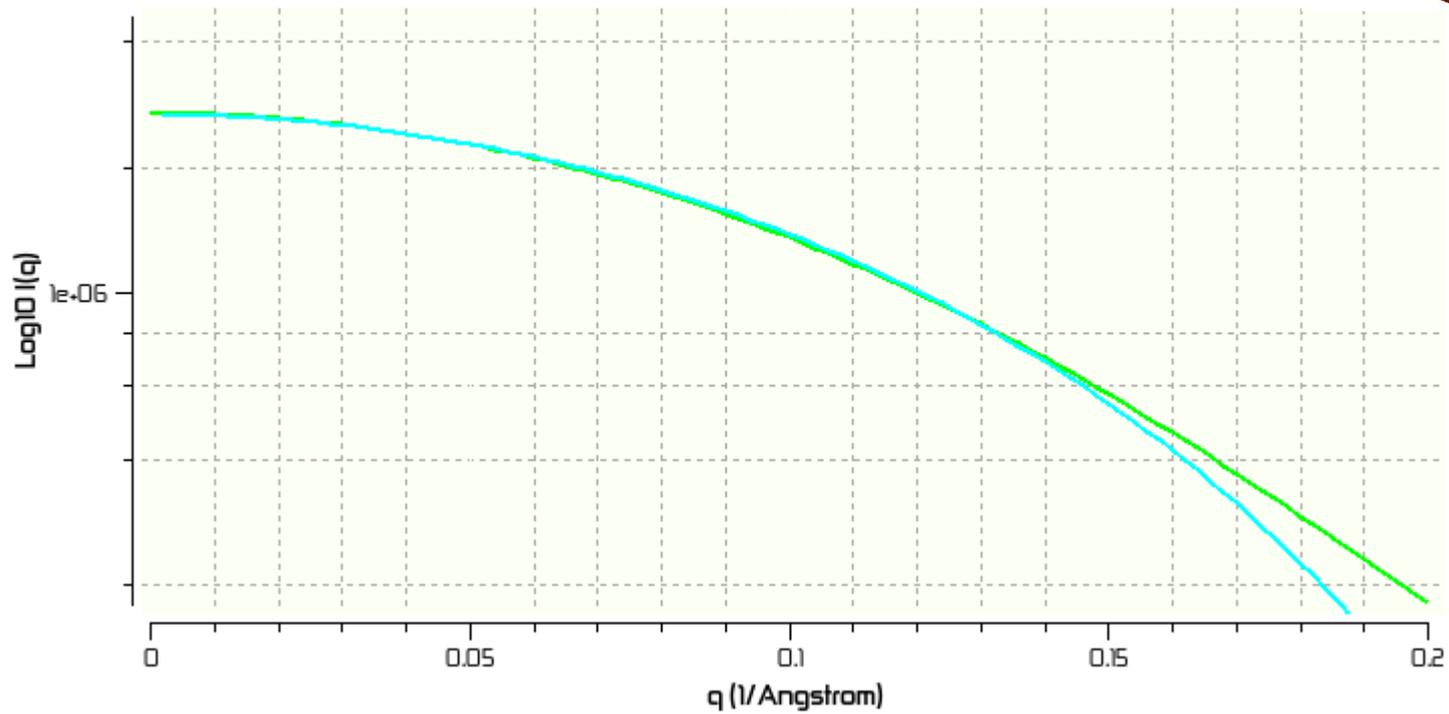
PDB	MW in Daltons	Description
8RAT.PDB	13,683.87	CRYSTALLOGRAPHIC STUDIES OF THE PROTEIN RIBONUCLEASE-A
1A4V.PDB	14,152.00	ALPHA-LACTALBUMIN
1DWR.PDB	17,682.20	MYOGLOBIN (HORSE HEART) WILD-TYPE COMPLEXED WITH CO
1HCO.PDB	32,279.78	HUMAN CARBONMONOXY HAEMOGLOBIN
1BEB.PDB	35,305.26	BOVINE BETA-LACTOGLOBULIN
1CTS.PDB	49,129.58	CITRATE SYNTHASE
2CGA.PDB	51,318.72	BOVINE CHYMOTRYPSINOGEN
1GZX.PDB	64,575.52	OXY T STATE HAEMOGLOBIN: OXYGEN BOUND AT ALL FOUR HAEMS
5LDH.PDB	74,917.32	ACTIVE TERNARY COMPLEX OF PIG HEART LACTATE DEHYDROGENASE WITH S-LAC-NAD
2GD1.PDB	144,427.77	OXIDOREDUCTASE(ALDEHYDE(D)-NAD(A))
1GD1.PDB	147,077.69	HOLO-GLYCERALDEHYDE-3-PHOSPHATE DEHYDROGENASE FROM BACILLUS STEAROTHERMOPHILUS
1ADO.PDB	157,287.20	FRUCTOSE 1,6-BISPHOSPHATE ALDOLASE FROM RABBIT MUSCLE
1OVA.PDB	169,965.56	UNCLEAVED OVALBUMIN

# Parsimonious modeling

1A4V - 1 sphere

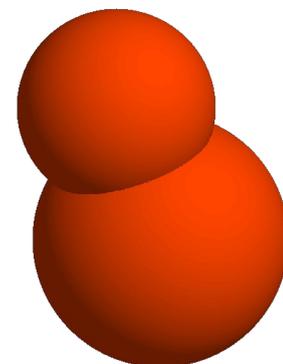
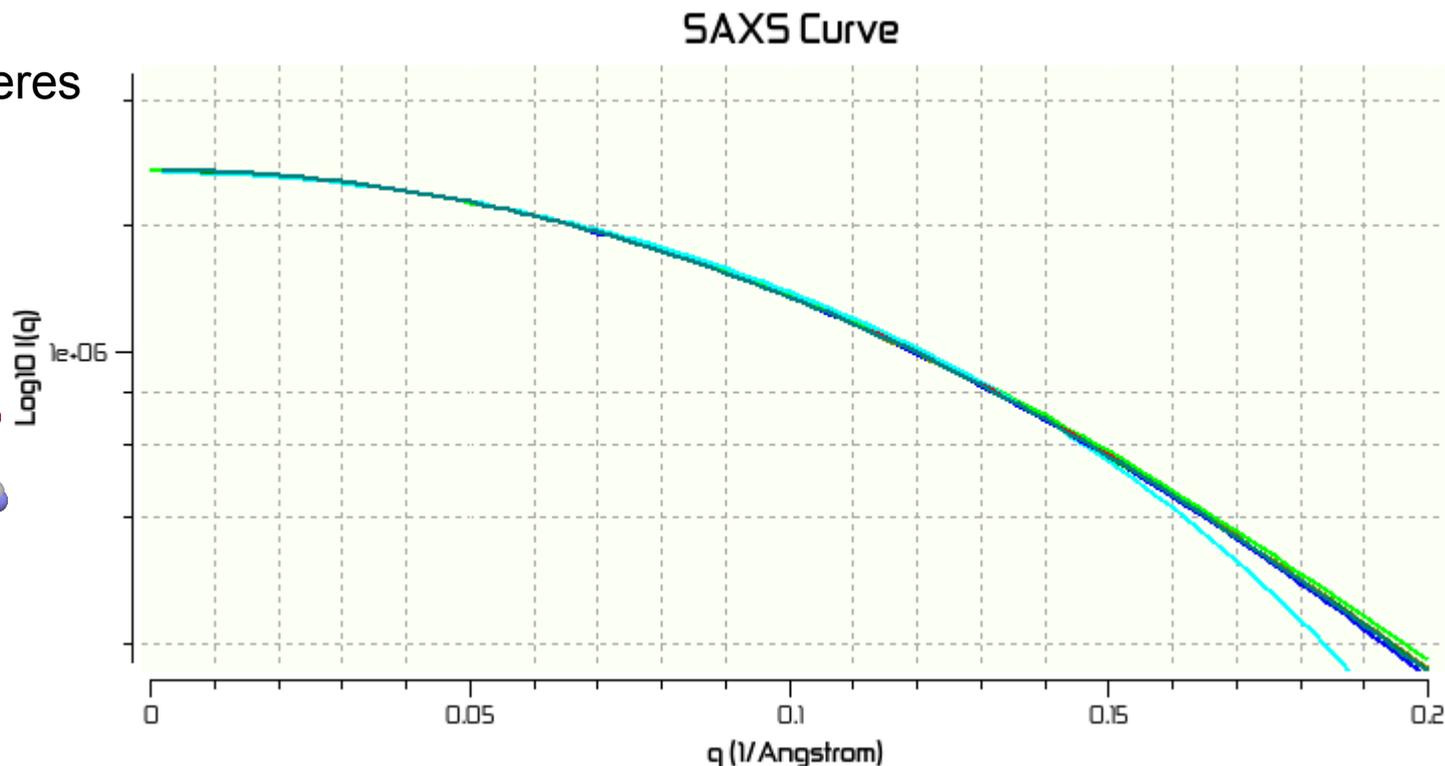
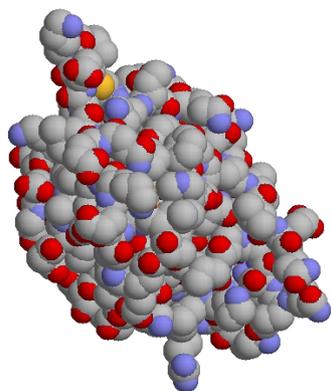


SAXS Curve



# Parsimonious modeling

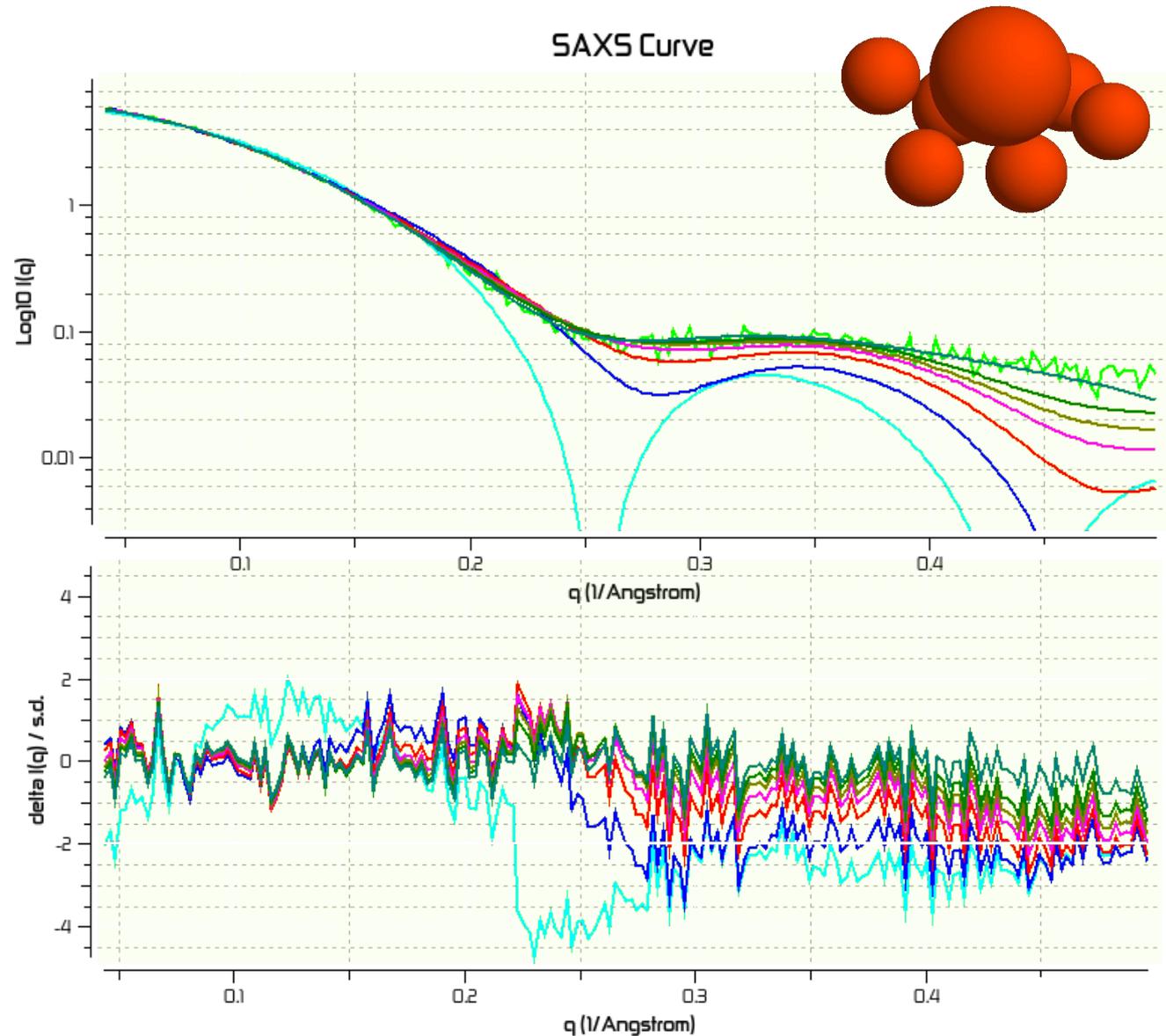
1A4V - 7 spheres



Model name	D(tr) [cm/sec <sup>2</sup> ]	Rg [nm]	Max extensions X [nm]	Y [nm]	Z [nm]	Axial ratios X:Z	X:Y	Y:Z
1A4V_1-db_1sa-10_1	1.210e-06	1.37	3.54	3.54	3.54	1.00	1.00	1.00
1A4V_1-db_2sa-10_1	1.200e-06	1.45	4.74	3.15	3.15	1.50	1.50	1.00
1A4V_1-db_3sa-10_1	1.140e-06	1.52	4.69	3.50	2.94	1.60	1.34	1.19
1A4V_1-db_4sa-10_1	1.170e-06	1.48	4.67	3.15	3.08	1.51	1.48	1.02
1A4V_1-db_5sa-10_1	1.140e-06	1.51	4.70	4.01	3.23	1.46	1.17	1.24
1A4V_1-db_6sa-10_1	1.140e-06	1.50	4.87	3.69	3.28	1.49	1.32	1.12
1A4V_1-db_7sa-10_1	1.160e-06	1.49	4.74	3.21	3.21	1.48	1.48	1.00
1A4V_1-so	1.137e-06	1.48	5.67	3.54	3.36	1.69	1.60	1.05

# Parsimonious modeling

- n-vector of spheres
- arbitrarily placed
- $I(q)$  reproduced with by ~7 spheres for test cases
- Physical extents reasonably matched
- overlap intensity issue
- connectivity



# *Parsimonious modeling*

- Extensible pool of parametrized shapes
  - Spheres, spheroids, ellipsoids, cylinders, tori
- Input  $I(q)$ , chosen shapes from pool
- → Parsimonious model

# Parsimonious modeling

## Parsimonious Spatial Modeling

Data files to process

Browse... lyzexp.dat

### Main model type controls

Model type list (0=sphere,1=cylinder,2=spheroid,3=ellipsoid,4=torus)

Compute models for all lengths upto above

Compute models for all unique combinations

Sample electron density [e/Angstrom<sup>3</sup>]

Buffer electron density [e/Angstrom<sup>3</sup>]

Grid size [Angstrom]

Automatically compute max

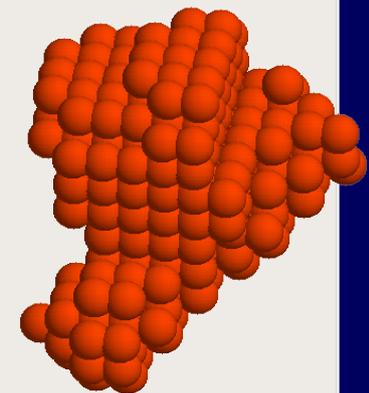
q range editing control

Supplementary control

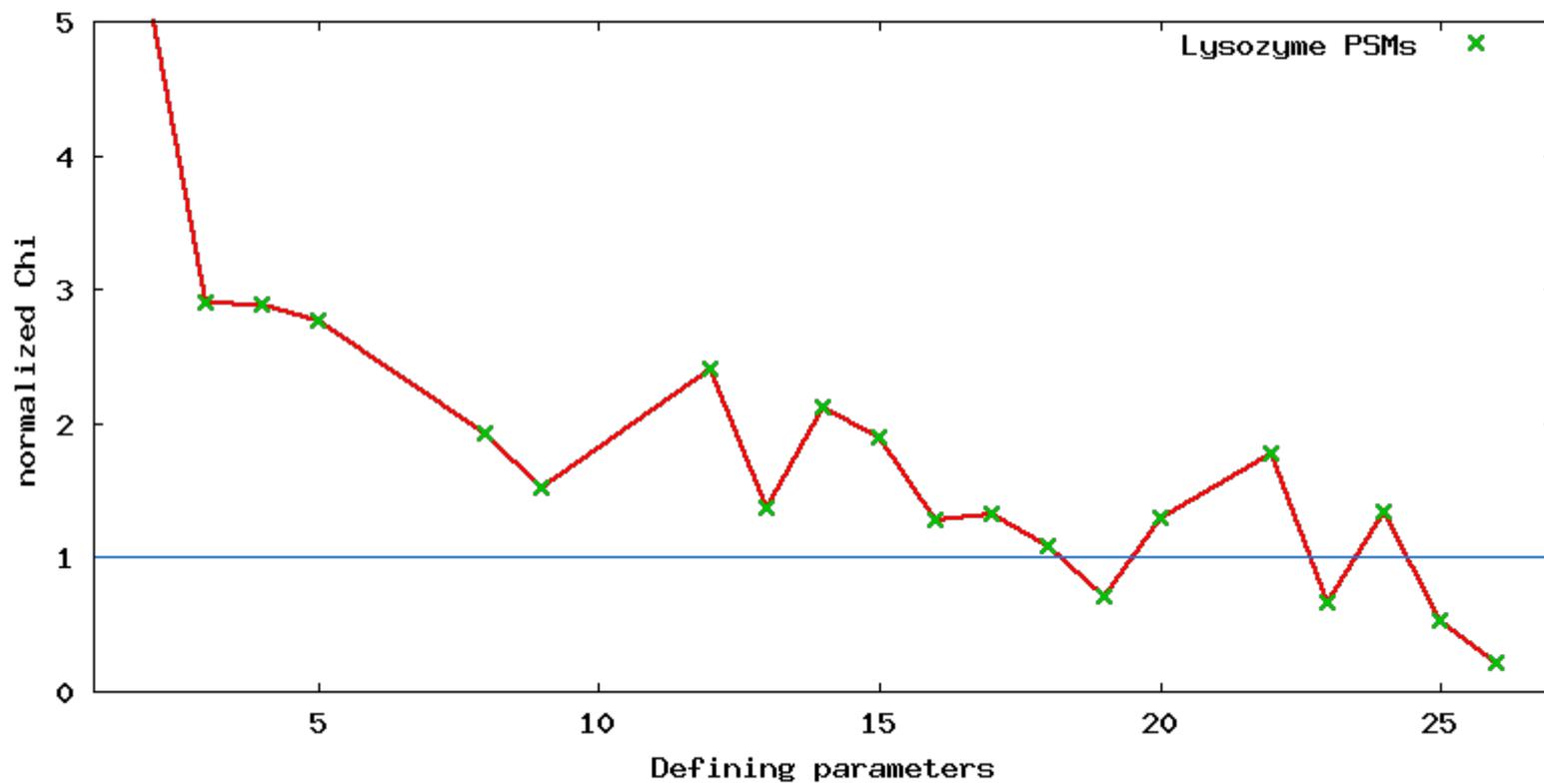
Genetic algorithm cont

Miscellaneous controls

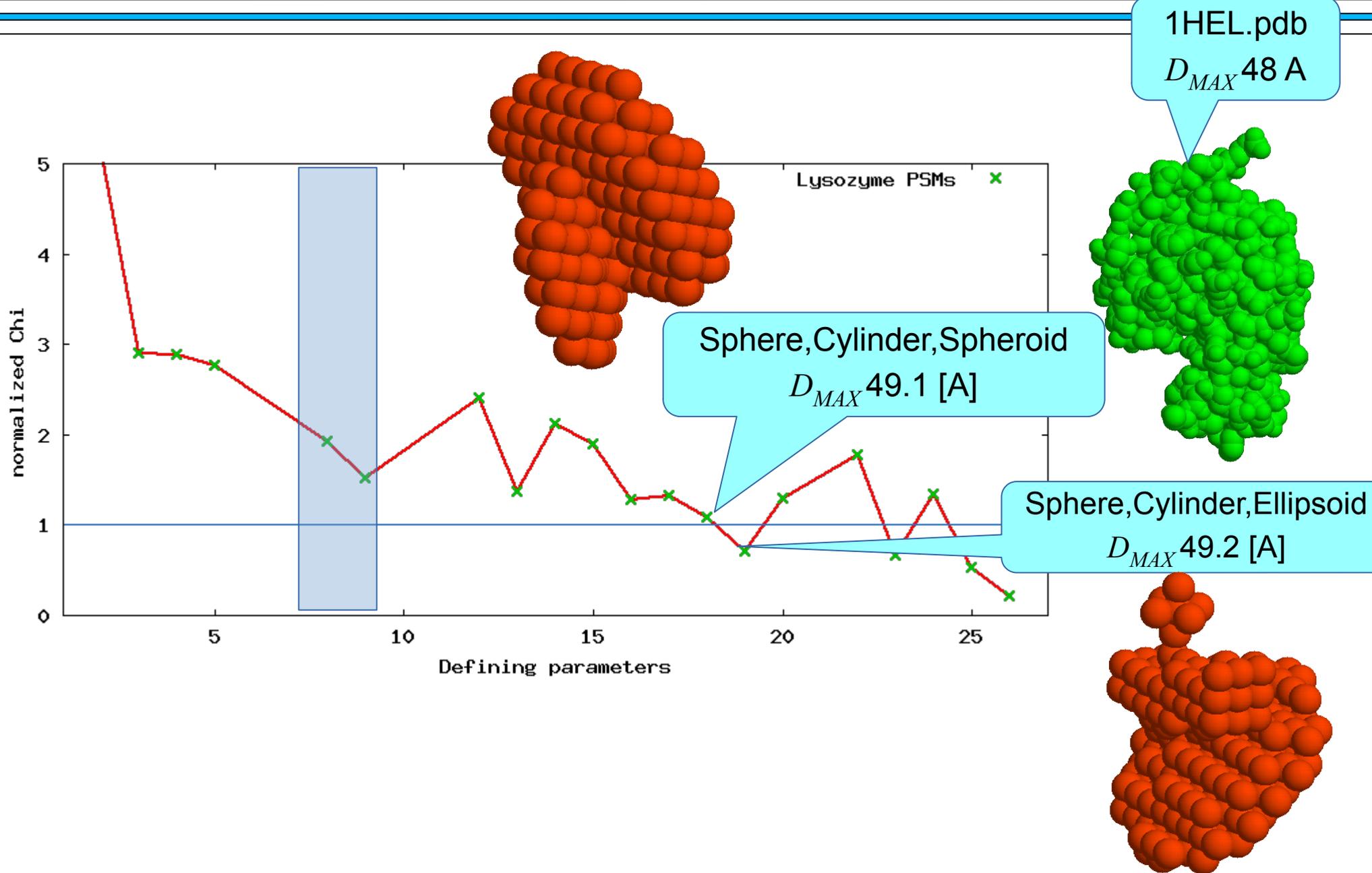
```
started
processing lyzexp.dat
computing approximate maximum dimension of lyzexp
approximate maximum dimension of lyzexp is 26
start GA for lyzexp_sphere
end GA for lyzexp_sphere model nchi 5.197
start GA for lyzexp_cylinder
end GA for lyzexp_cylinder model nchi 3.137
start GA for lyzexp_sphere_sphere
end GA for lyzexp_sphere_sphere model nchi 2.769
start GA for lyzexp_sphere_cylinder
end GA for lyzexp_sphere_cylinder model nchi 2.794
start GA for lyzexp_cylinder_cylinder
end GA for lyzexp_cylinder_cylinder model nchi 2.458
start GA for lyzexp_sphere_sphere_sphere
end GA for lyzexp_sphere_sphere_sphere model nchi 1.915
start GA for lyzexp_sphere_sphere_cylinder
end GA for lyzexp_sphere_sphere_cylinder model nchi 2.341
start GA for lyzexp_sphere_cylinder_cylinder
end GA for lyzexp_sphere_cylinder_cylinder model nchi 1.097
start GA for lyzexp_cylinder_cylinder_cylinder
end GA for lyzexp_cylinder_cylinder_cylinder model nchi 0.3411
start GA for lyzexp_sphere_sphere_sphere_sphere
```



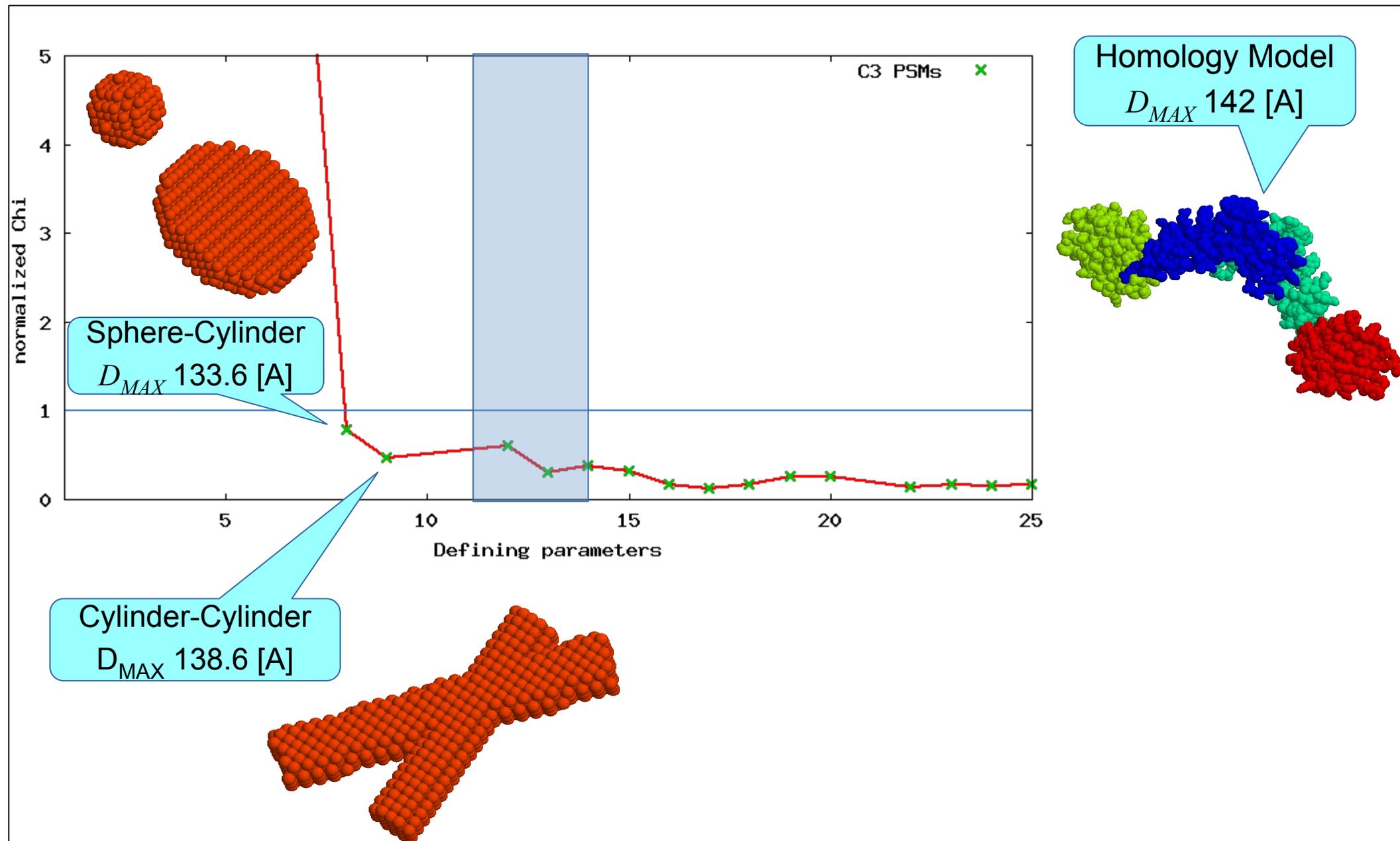
# Parsimonious modeling



# Parsimonious modeling

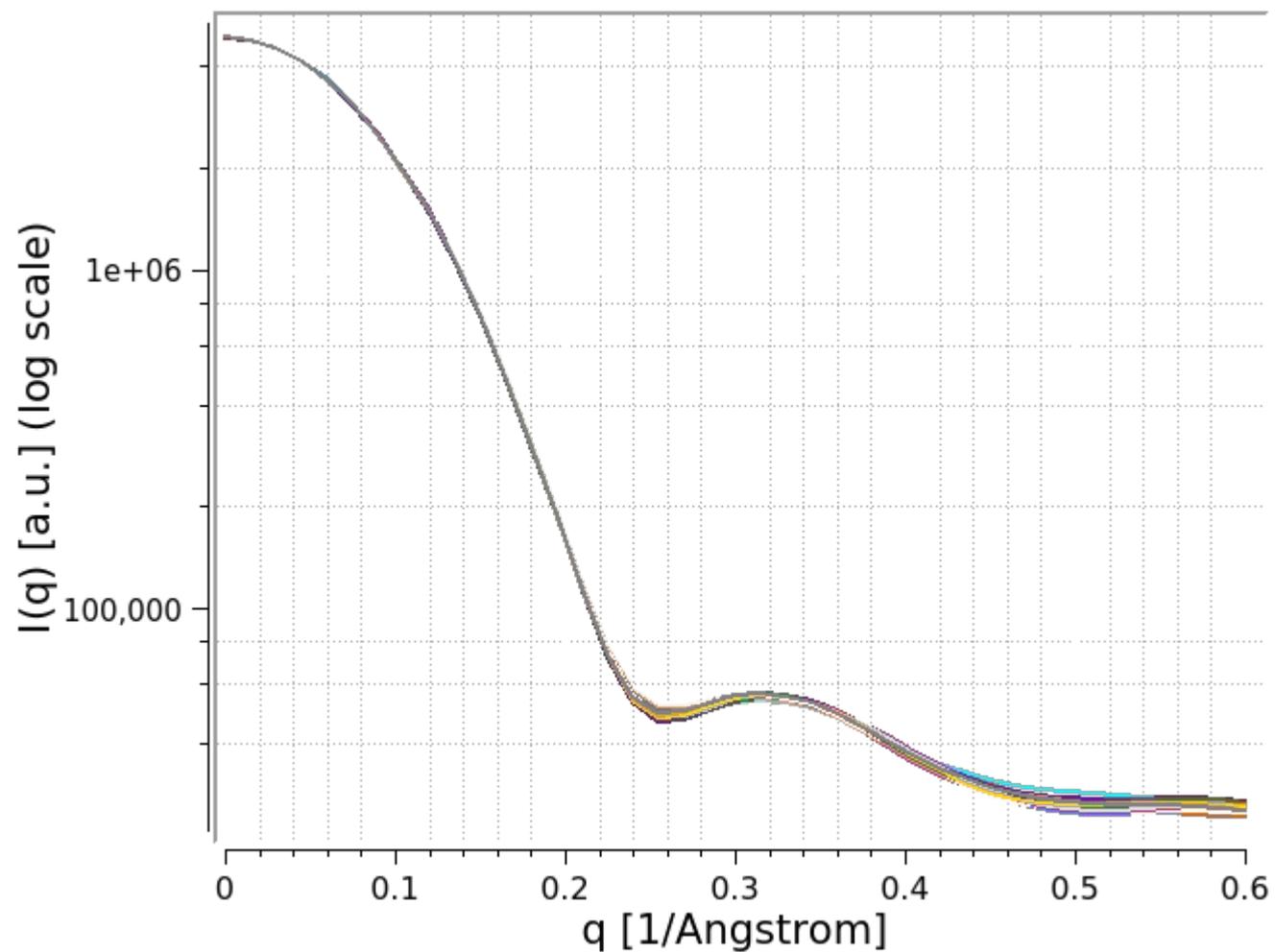


# Parsimonious modeling



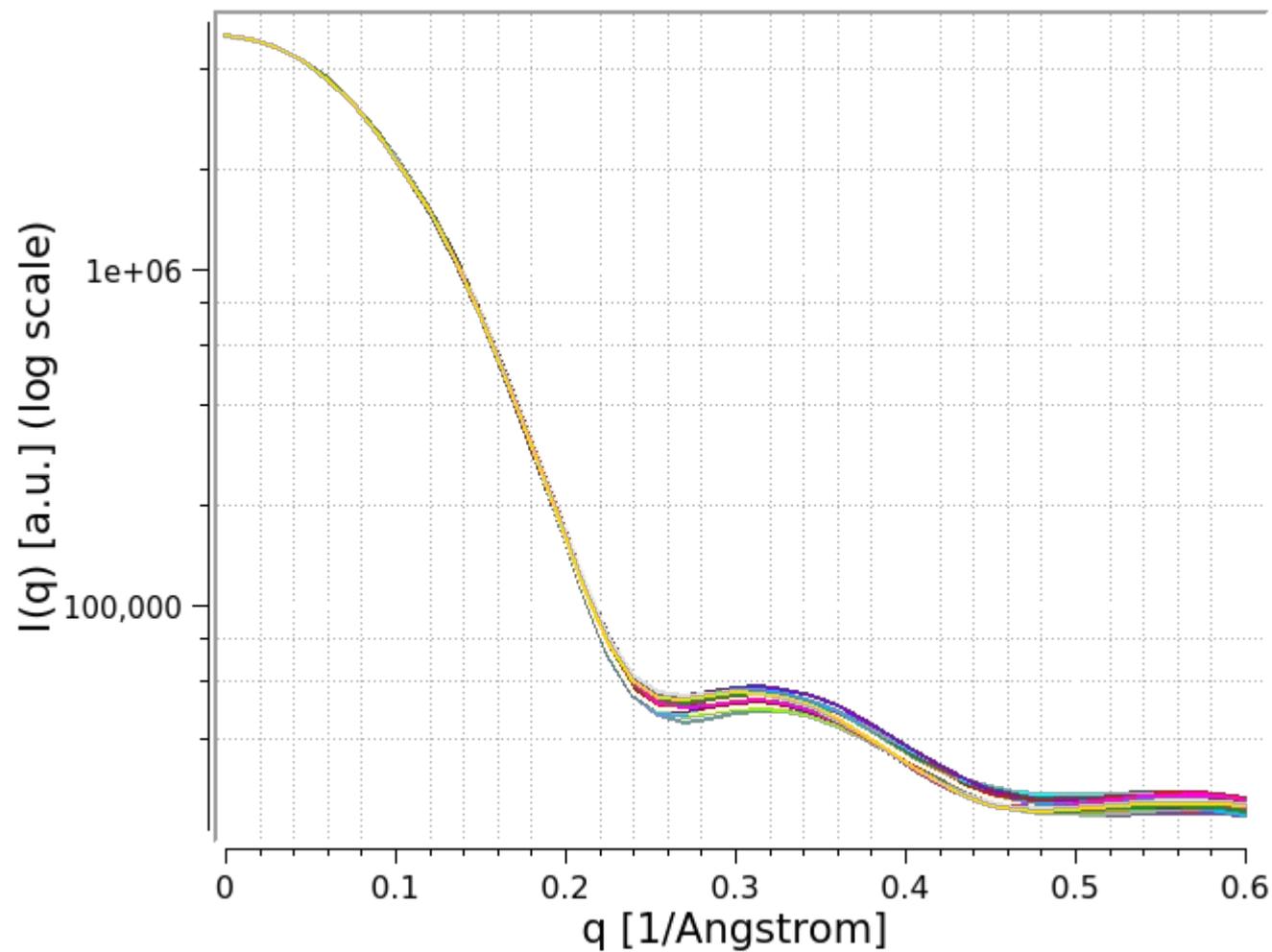
# *SAS is very sensitive*

1HEL  
50 frames DMD  
10 ps/frame



# *SAS is very sensitive*

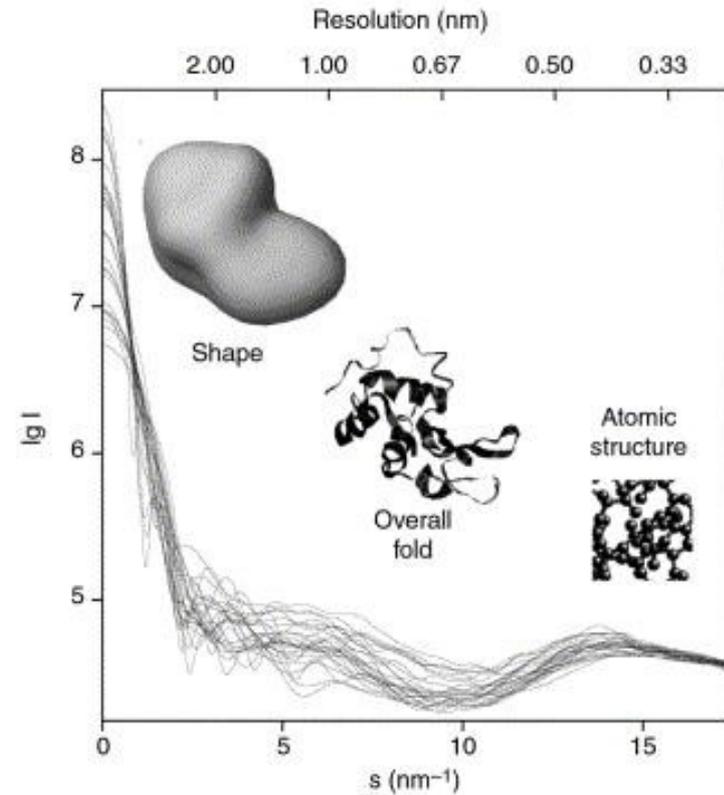
1AKI  
48 frames NAMD  
100mM NaCl  
10 ps/frame



# Information in a SAS curve

Low IC:  $D_{max} \cdot q\text{-range} / \pi$

High sensitivity

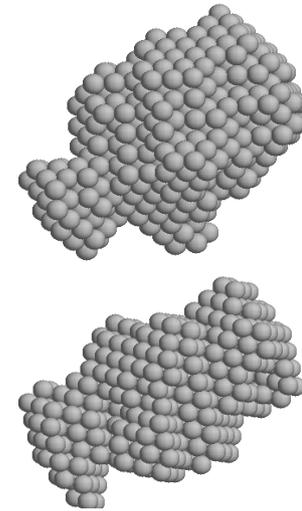
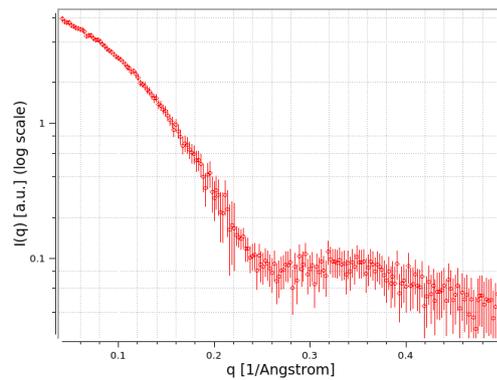


GFDL, <https://en.wikipedia.org/w/index.php?curid=13264733>

# Ab-initio reconstruction

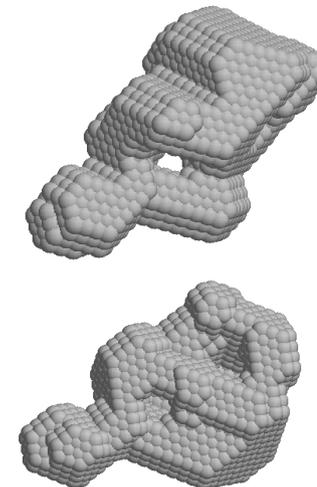
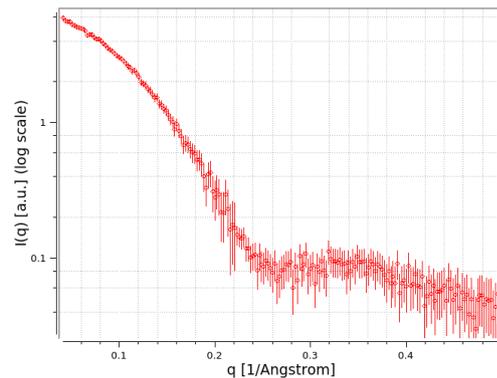
## DAMMIN

*D. I. Svergun (1999) Biophys J. 2879-2886.*



## DAMMIF

*Franke, D. and Svergun, D.I. (2009) J. Appl. Cryst., 42, 342-346.*

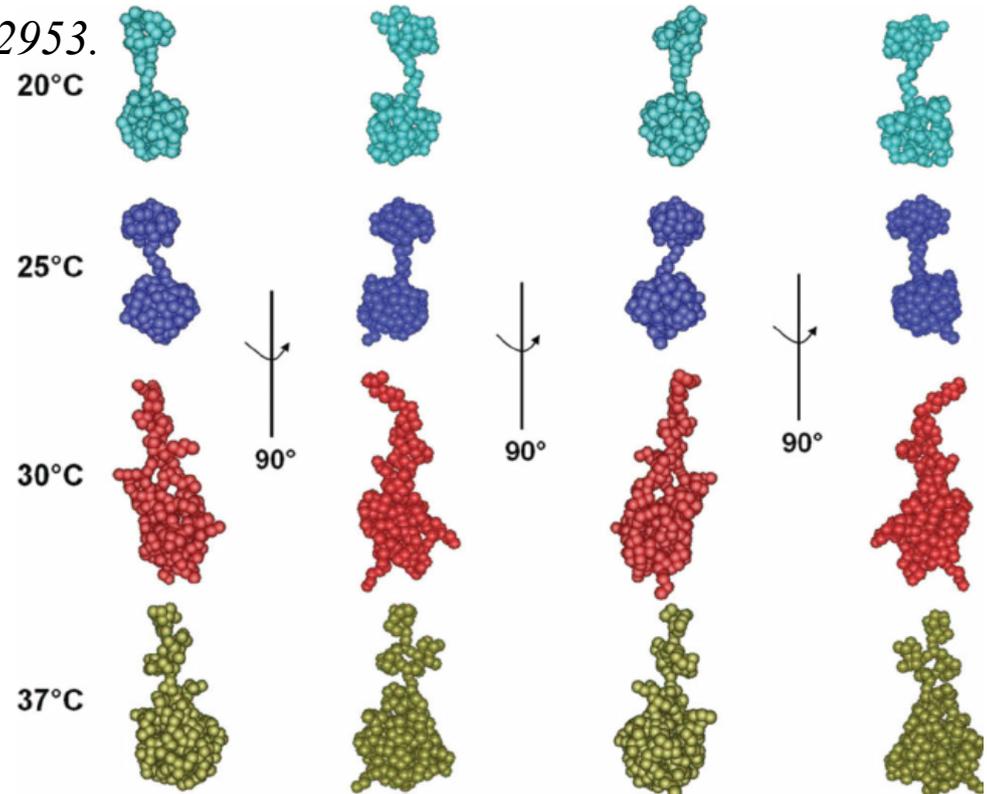


# Ab-initio reconstruction

## GASBOR

*Svergun, D.I. et al. (2001) Biophys. J., 80, 2946-2953.*

ab initio reconstruction of protein structure  
by a chain-like ensemble of dummy residues



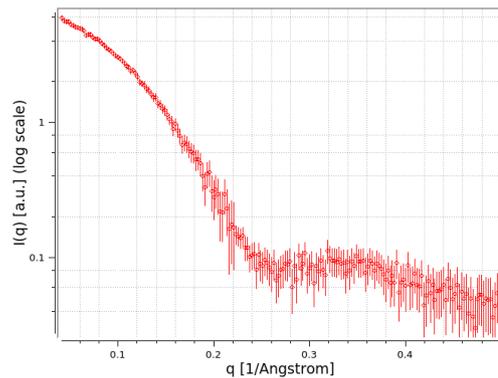
Various views of the ab initio 3D models obtained using GASBOR and by averaging ten single models for each sample by using DAMAVER for rSdrFB1-4 at different temperatures.

*Dipoto, Antonella et al., (2015). Appl. Microbiology and Biotech.*

# Ab-initio reconstruction

## DENSS

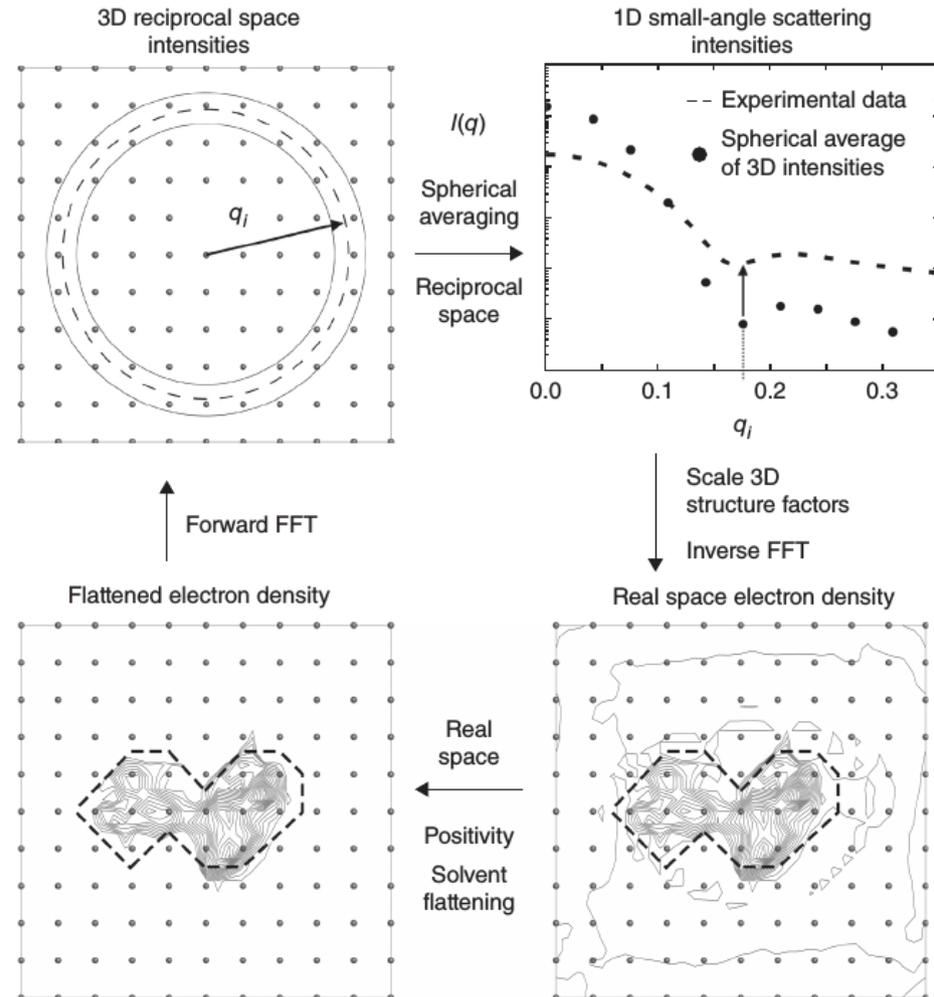
*T. D. Grant (2018) Nat. Meth. 15:3 191-193*



$$f(\mathbf{q}) = \int_{V_r} \rho(\mathbf{r}) dV_r$$

$$\mathbf{F}(\mathbf{q}) = \sum_{i=1}^N f_i \mathbf{e}^{i\mathbf{r}_i \cdot \mathbf{q}}$$

$$\mathbf{I}(\mathbf{q}) = \mathbf{F}(\mathbf{q}) \cdot \mathbf{F}^*(\mathbf{q})$$



As with DAMMIN, DAMMIF, GASBOR →stochastic, averaging of multiple models

# Information content revisited

*Jochen S Hub. Curr. Op. in Struct. Bio. 2018, 49:18-26*

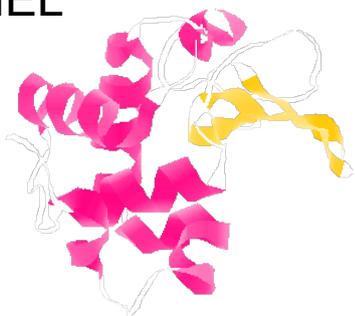
“the interpretation of solution scattering data by computational methods is complicated by the low information content of the data, by scattering contributions from the hydration layer, and by unknown systematic errors.”

“The physical information in atomistic force fields complements the low-information SWAXS data; explicit-solvent MD may be used to predict solvent scattering, and the MD-related sampling methods may guide the structure refinement against SWAXS data.”

“Because SWAXS curves are smooth and one-dimensional (1D), they contain quite a limited amount of information. How the information is distributed over the  $q$ -range is a matter of ongoing research, but it is generally accepted that experimental SWAXS curves do not contain more than 10–30 independent data points. Hence, the number of backbone angles of biomolecules exceeds the number of independent data points of SWAXS curves by roughly two orders of magnitude. This precludes any straightforward fitting of protein structures against SWAXS data, but instead it leads to a high risk of overfitting.”

# Comparing models against data

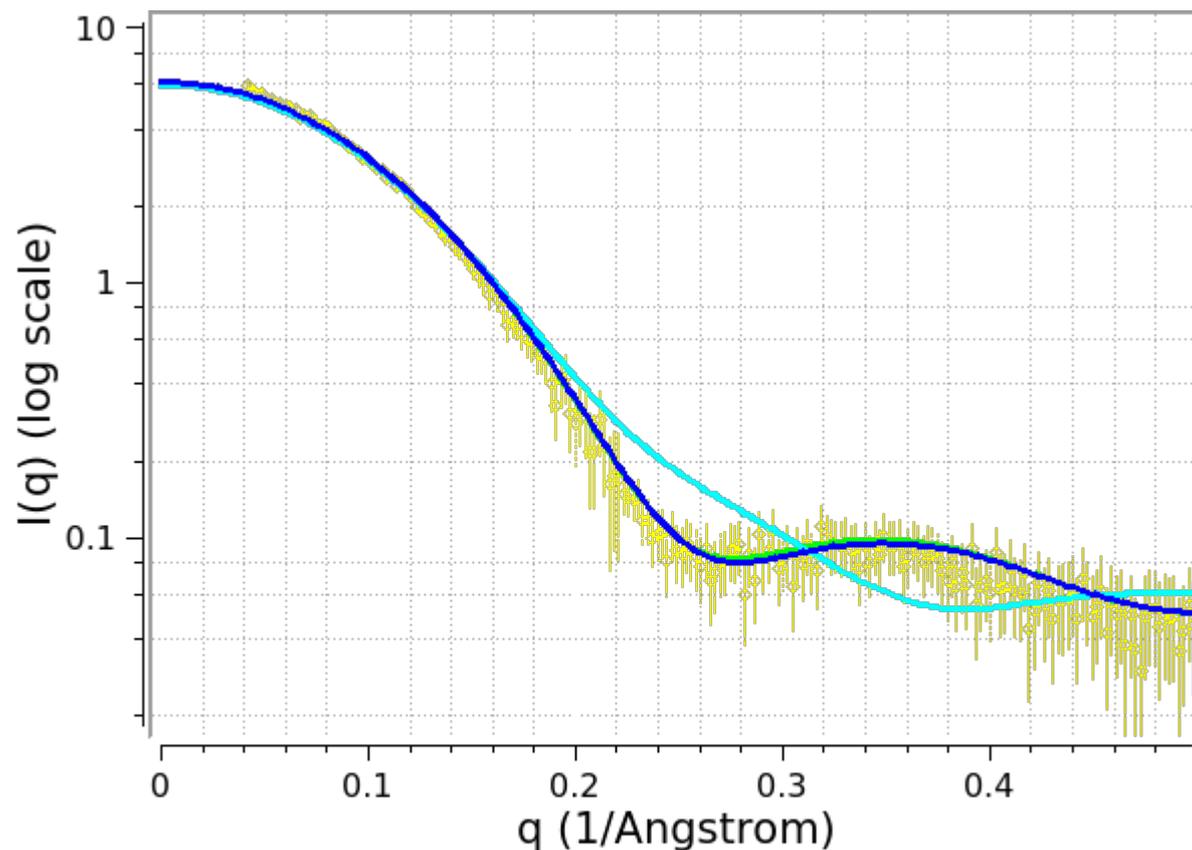
1HEL



8RAT



1AKI



lyzexp.dat

1HEL-cr\_h15\_g17\_hs0.int

8RAT-cr\_h15\_g17\_hs0.int

1AKI-cr\_h15\_g17\_hs0.int

Sorted best fit results

0.914138 : "1HEL-cr\_h15\_g17\_hs0\_03.int"

0.922224 : "1AKI-cr\_h15\_g17\_hs0\_03.int"

1.2017 : "8RAT-cr\_h15\_g17\_hs0\_03.int"

Best fit model: "1HEL-  
cr\_h15\_g17\_hs0\_03.int" nchi 0.914138

NNLS results:

1AKI-cr\_h15\_g17\_hs0\_03.int 0

1HEL-cr\_h15\_g17\_hs0\_03.int 0.726835

8RAT-cr\_h15\_g17\_hs0\_03.int 0.273165

Scaling factor: 1  $\chi^2=143.506$   $df=195$   
nchi=0.857863

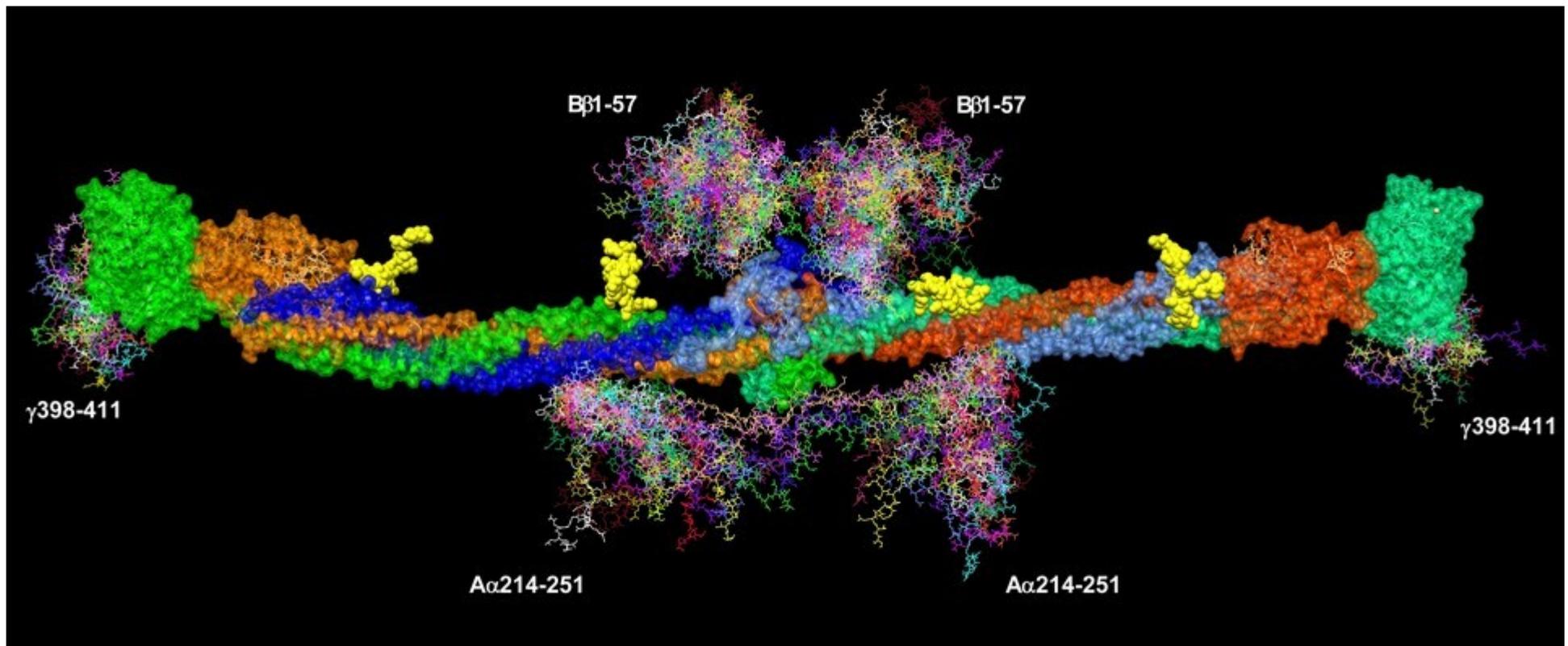
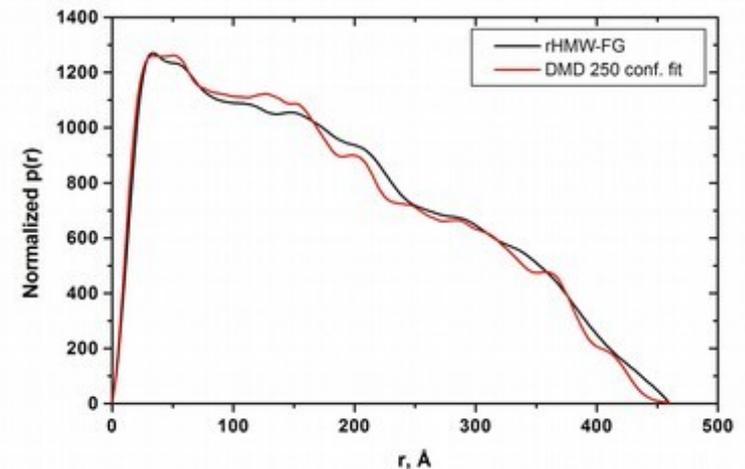
# Comparing models against data

Mattia Rocco et al.

Fibrinogen is an important component of the coagulation cascade, as well as a major determinant of blood viscosity and blood flow

A centrosymmetric dimer made by 3 pairs of chains

US-SOMO/DMD simulations of the conformational variability for comparison to experimental data



*Images: Mattia Rocco*

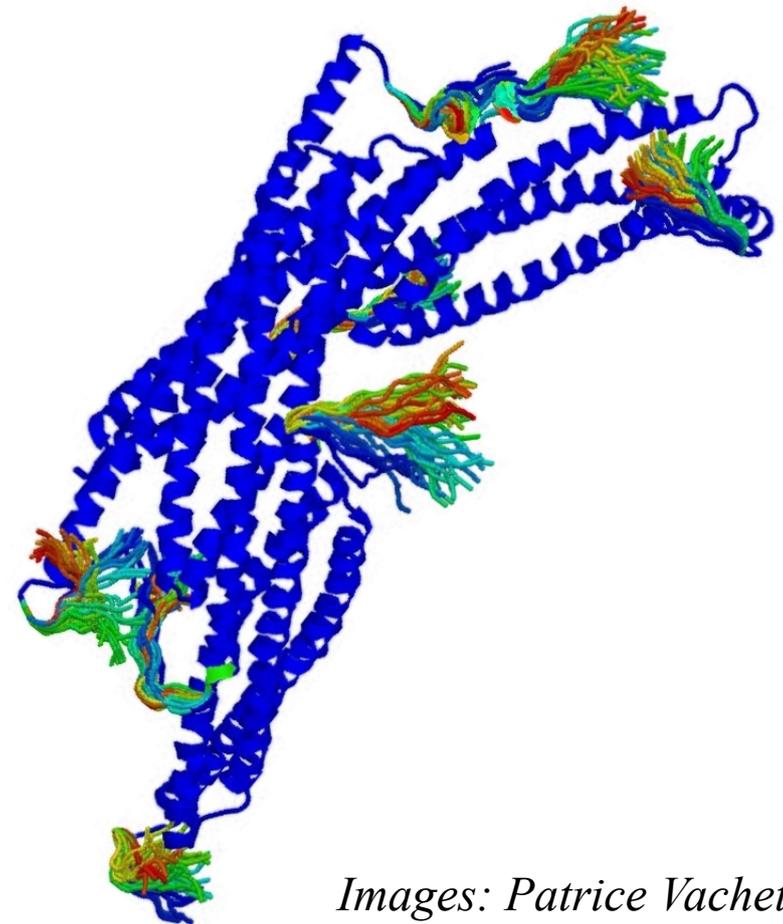
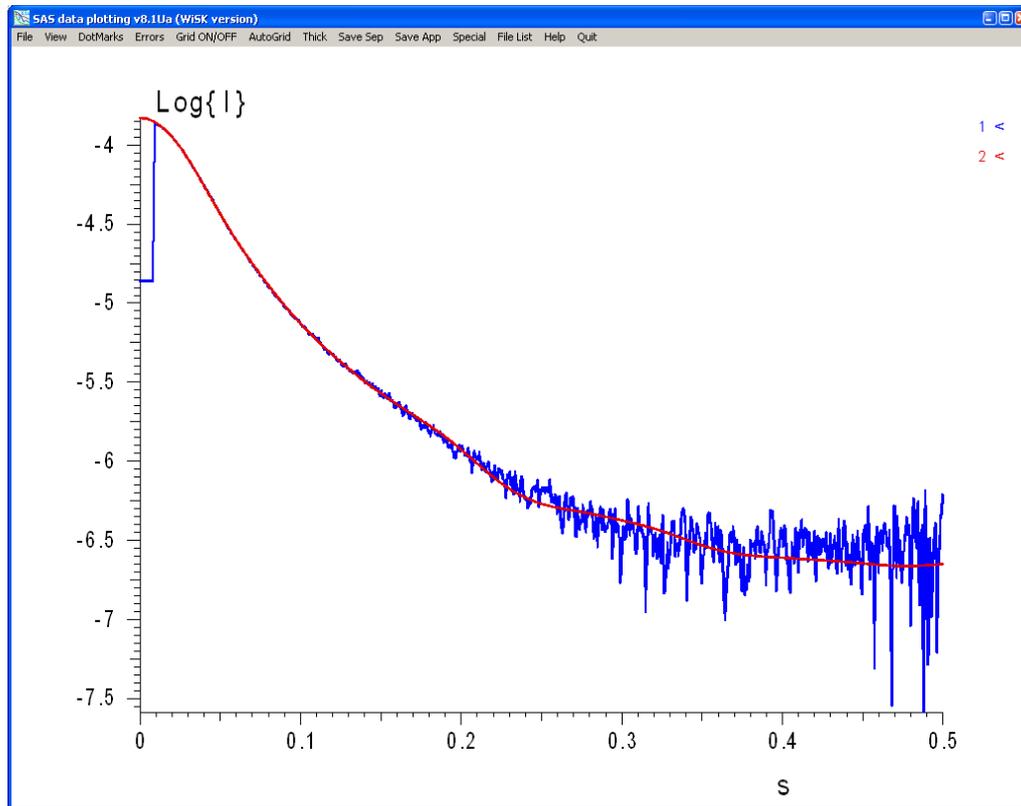
# Comparing models against data

Group of M. Milani, University of Milano

Smac-DIABLO a dimeric protein involved in apoptosis (programmed cell death)

chain: 192 residues, MM= 21.8 kDa

Final refinement of model by addition of N- and C-termini using US-SOMO/DMD



Images: Patrice Vachette

# Comparing models against data

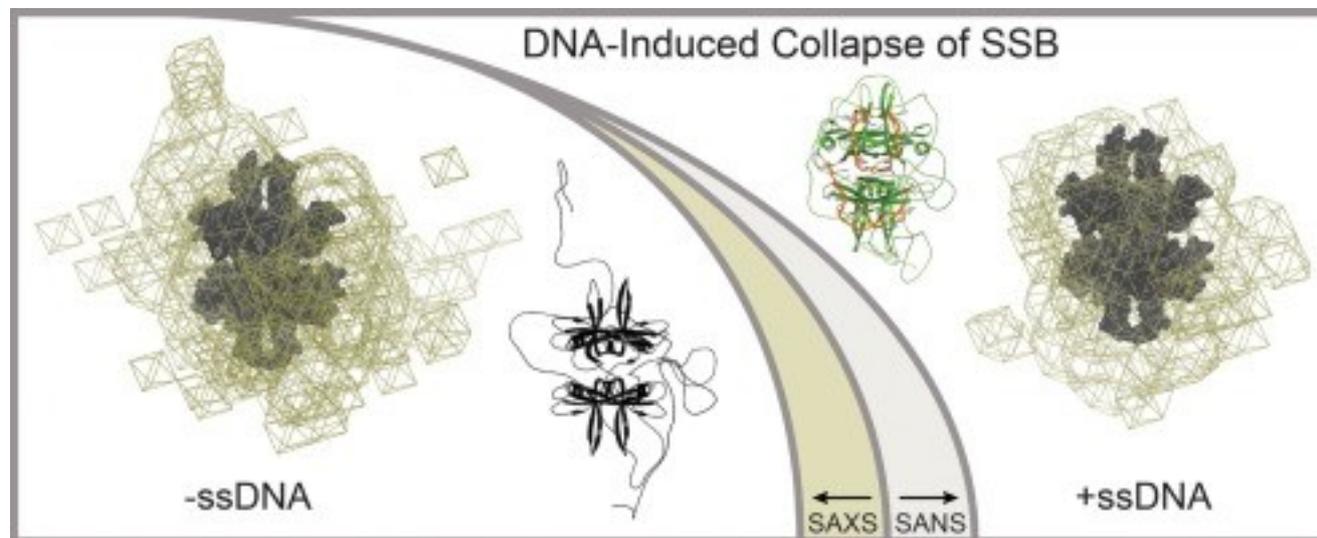
Mathew Green et al, University of Nottingham

SSB binds individual strands of DNA

Critical role in DNA metabolism: Replication, recombination & repair

Intrinsically disordered

US-SOMO/DMD used to create conformations to screen against SAXS data



# *Expanding conformational space*

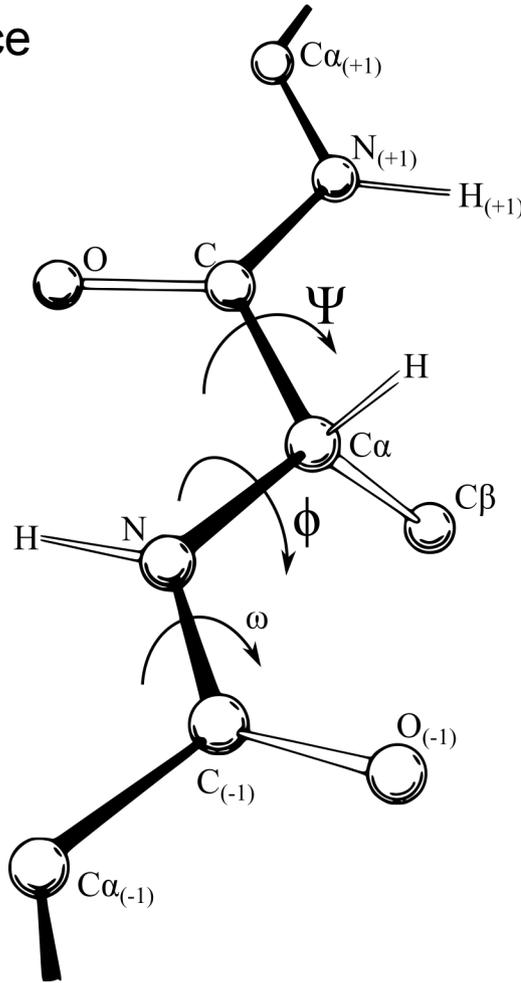
MD – frequently very high computational demand to cover conformational possibilities without restraints

MC – can cover conformational space faster

Often not accessible to a biologist without a steep learning curve...  
convenience tools are available to help.

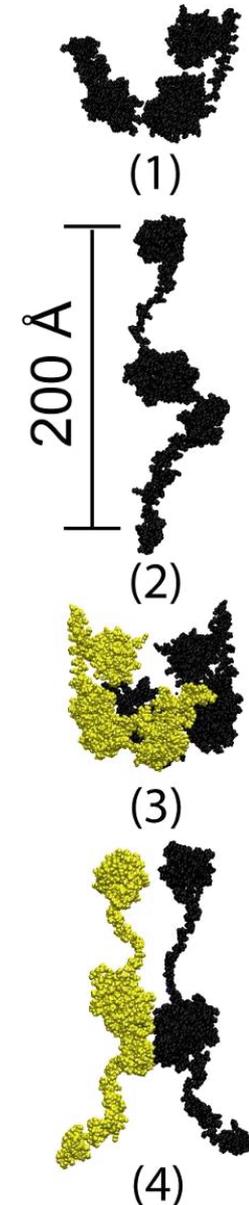
# Expanding conformational space – Dihedral angles

Expanding conformational space  
Dihedral angle MD & MC



By Dcrjrs, vectorised Adam Rędzikowski - Own work, CC BY 3.0,  
<https://commons.wikimedia.org/w/index.php?curid=24585750>

HIV-1 Gag



Siddhartha A.K. Datta et al.,  
*J Mol Biol.* 2011; 406(2): 205–214.

# Expanding conformational space - SASSIE

Developed to enable NCNR user community to efficiently develop molecular models for the neutron/X-ray scattering/reflectivity experiments.

[www.smallangles.net/sassie](http://www.smallangles.net/sassie)

### Monomer Monte Carlo

run name:

reference pdb:  hiv1\_gag.pdb OR  Local: hiv1\_gag.pdb

output file name (dcd):

number of trial attempts:

return to previous structure:

temperature (K):

molecule type:

number of flexible regions to vary:

maximum angle sampled for each region:

residue range for each flexible region:

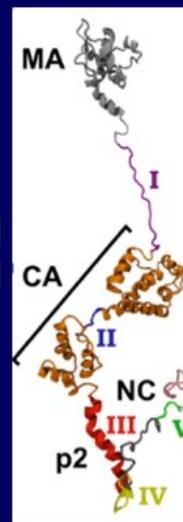
structure alignment: low residue:

structure alignment: high residue:

overlap basis:

Advanced Input

Check Box for Advanced Input:



Domain	Flexible Region	Residues
MA		1 - 122
linker	I	123 - 144
CA		145 - 276
	II	277 - 282
		283 - 353
p2	III	354 - 377
linker	IV	378 - 389
NC		390 - 407
	V	408 - 412
		413 - 432



**NIST**  
National Institute of Standards and Technology  
Technology Administration, U.S. Department of Commerce

Generate Ensemble

Energy Minimize  
&  
Calculate  $I(q)$

Compare to Exp.:  
Chi-square Filter  
Density Plot

# Rigid body modeling

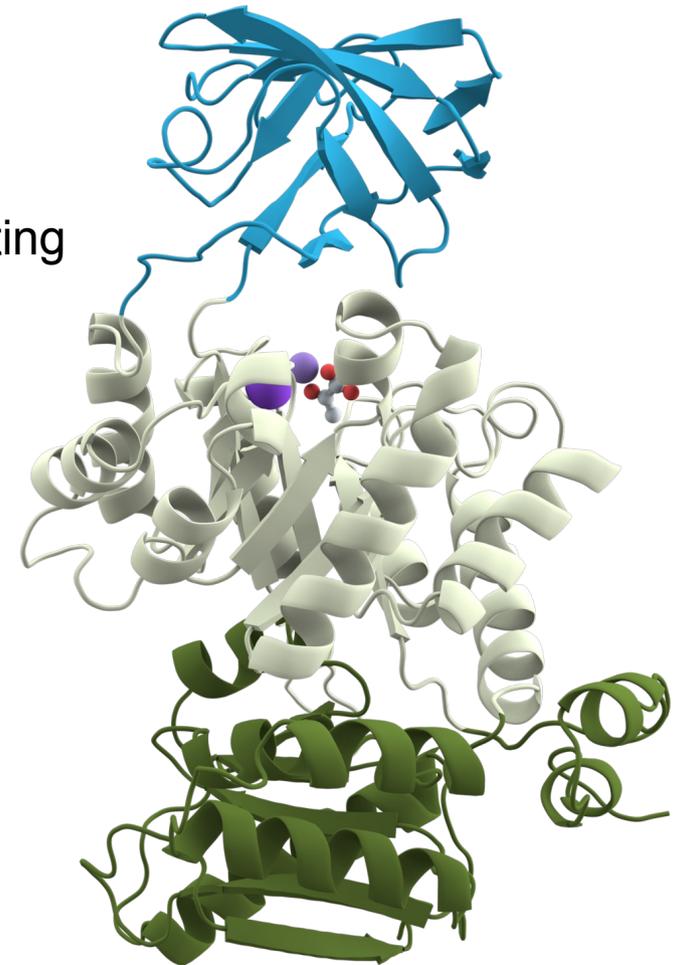
Structure of subunits are known

Arbitrary complex can be constructed by moving and rotating

Verify no steric clashes

→ scattering data subunits

- + contacts (chemical shifts by NMR or mutagenesis)
- + distances between residues (FRET or mutagenesis)
- + relative orientation (RDC by NMR)



Pyruvate kinase 1PKN

By Thomas Spletstoesser ([www.scistyle.com](http://www.scistyle.com)) -  
Own work, CC BY-SA 3.0

# Software landscape

Software for “data reduction”, “visualization”, “model fitting”, various “analysis” ...  
Grouped packages and stand alone components

ATSAS – Dmitri Svergun group

Scatter – Rob Rambo

BioXTAS Raw – Jesse Hopkins

SASView – multiple contributors

CCP-SAS – SCT/SCTPL / US-SOMO / SASSIE & others – multiple contributors

more at <http://smallangle.org/content/software>

Table 1

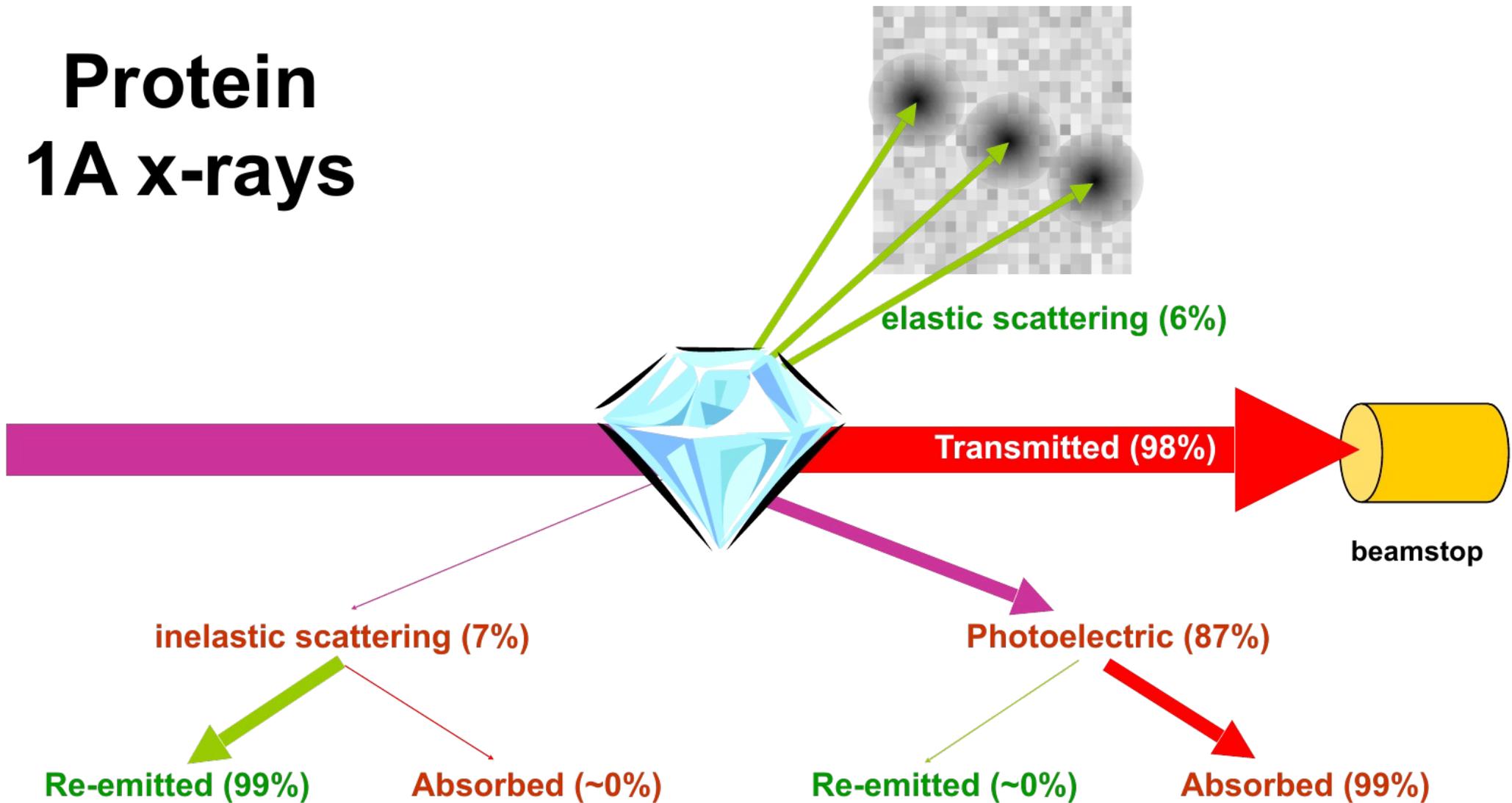
Incomplete list of methods for predicting SWAXS curves from structural models: Fitting of hydration layer required ( $\delta\rho_{\text{fit}}$ , including method that ignore the hydration layer), using tabulated reduced form factors ( $f_{\text{red}}$ ), resolution [atomistic or coarse grained (CG)], fluctuations included, free availability [Download (D), web server (W)]. Additional software is listed in Refs. [63,64]

ID	Name/authors	Year	$\delta\rho_{\text{fit}}/f_{\text{red}}$	Resol.	Fluct.	Avail.	Refs.
<b>Implicit solvent methods</b>							
1	CRY SOL	1995	Yes/yes	atom.	–	D/W	[25]
2	ORNL-SAS	2007	Yes/yes	atom.	–	D	[65]
3	SoftWAXS	2009	Yes/–	atom.	–	D	[66]
4	Fast-SAXS-pro	2009	Yes/yes	CG	Yes	D/W	[30,36]
5	FoXS	2010	Yes/yes	atom.	–	D/W	[67,29]
6	PHAISTOS	2010	Yes/yes	CG	–	D	[68]
7	AquaSAXS/AquaSol	2011	Yes/yes	atom.	–	W	[27]
8	SASbtX/Zernike	2012	Yes/–	atom.	–	W	[69]
9	RISM-SAXS	2014	–/yes	atom.	–	D	[70]
10	BCL::SAXS	2015	Yes/yes	atom.	–	D	[71]
11	Pepsi-SAXS	2017	yes/yes	atom.	–	D	[72]
<b>Explicit solvent methods</b>							
12	SASSIM/Sassena	2002	–/yes	atom.	Yes	D	[73]
13	MD-SAXS	2009	–/–	atom.	Yes	–	[74,75]
14	AXES	2010	Yes/–	atom.	–	W	[26]
15	HyPred	2011	–/–	atom.	–	W	[76]
16	Park et al.	2009	–/–	atom.	–	–	[77]
17	Köfinger & Hummer	2013	–/–	atom.	Yes	D	[78]
18	WAXSiS	2014	–/–	atom.	Yes	D/W	[38,79]

Table from Jochen S Hub. *Curr. Op. in Struct. Bio.* 2018, 49:18-26

# Where do Photons go?

## Protein 1A x-rays



James Holton

# ***Practical considerations***

## **Sample requirements for (SAXS) solution scattering**

- size: >5kD
- purity: highly monodisperse !
- concentration: 0.25 – 10mg/ml (higher for small proteins and intermediate angle data)
- sample volume 15-50 ul ;(so only a fraction of 1mg protein needed for a starting experiment!)
- enough material for at least 3 concentrations
- matching buffer solution is very important (lower salt better)
- most buffer components tolerated (e.g. glycerol (<30%) and salt (<0.5M) are OK)
- S-reducing agent can help protein to stay intact under irradiation

### Additional requirements for time-resolved measurements

- lots of sample (at least 10mg, better more)
- sufficiently large change between initial and final state
- pre-characterization of kinetics by other techniques

# ***Practical considerations***

## **A good SAXS experiment starts in your home lab**

- every protein has its own “personality”
  - the more you know about your protein the better you can select the data acquisition parameters (buffer composition, pH, additives ....)
- Characterize your protein as much as possible with biochemical means
  - check for possible oligomerization with concentration
    - in case of complexes: for dissociation under dilution
  - determine highest concentration the protein is stable (and how long?)
  - simulate shipping conditions (e.g. freezing & thawing) and check sample quality afterwards
- know your numbers
  - sequence and MW
  - extinction coefficient and concentration of your stock solution

# Practical considerations

## Monodispersity

- check your samples:
  - Good solubility (clear solution), no obvious precipitates
  - Single species on native gels
  - SDS-PAGE should show no contamination
  - Single symmetric peak on an SEC column
- Other analytical techniques:
  - Dynamic light scattering (DLS)
  - Analytical ultracentrifugation
  - Mass spectrometry SEC-MALLS

## Buffer conditions

- use a low salt concentration if possible
- for proteins PBS buffer is usually a good choice
- consider additives to prevent radiation damage (DDT, TCEP, Glycerol ...)
- bring plenty of matched buffer

# ***Practical considerations***

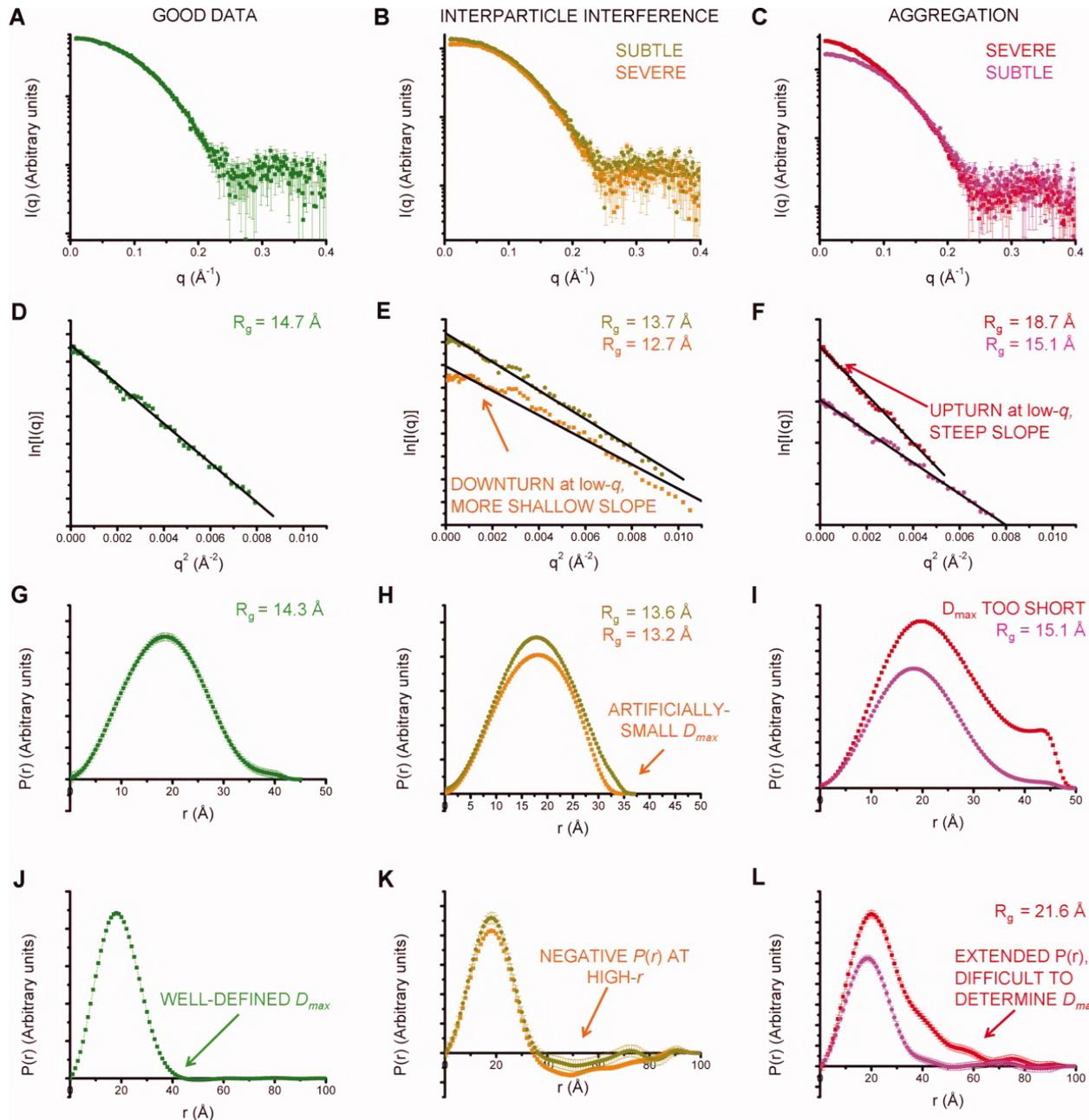
## **Before coming to SSRL**

- provide accurate information in the beamtime request form
- ask beamline staff if you are unsure or have questions
- contact your beamline staff before experiment just in case something changed

## **At the beamline**

- understand how the data collection works and how to load your samples
- take plenty of buffer images
- take advantage of the online data reduction: **monitor what's happening!**
- consider sample recovery for post exposure analysis
- bring additional radical scavengers in case of unexpected radiation damage

# Practical considerations – Data Quality



## Immediate data quality checks

- aggregation:
  - upturn at low  $q$
  - residuals in guinier plot will show upward curvature
- interparticle repulsion:
  - downturn at low  $q$
  - residuals in guinier plot will show downward curvature
  - will increase with concentration

## Checks with the $p(r)$ function

- determine  $D_{max}$ 
  - no “nose-diving” !
  - no excessive oscillation around 0
  - rule of thumb:  $D_{max} \approx 3 \cdot R_g$
  - Switch off  $P(d_{max})=0$  and use large  $D_{max}$  to estimate
- determine  $R_g$ 
  - should compare well with  $R_g$  from Guinier

## **What if your Sample is Aggregated?**

- centrifuge your sample (ideally keep it cold)
- dilute and centrifuge
- filter
- add more DTT if radiation damage is the problem
- run sample through SEC column if time permits
- change buffer condition (if you have enough material)

# Practical considerations – Finding a SAXS beamline



## Light sources of the world

There are more than 50 light sources in the world (operational, or under construction). This page lists all the members of the lightsources.org collaboration.



 Orange pins on the map represent members of the lightsources.org collaboration.

# Practical considerations – Finding a SAXS beamline



APS at Argonne National Laboratory



BESSY II at HZB



Elettra Sincrotrone Trieste



NSLS-II at Brookhaven National Laboratory



Pohang Light Source-II



PETRA III at DESY



SESAME  
(Honorary Member)



SSRL at SLAC



SYNCHROTRON  
THAILAND  
CENTRAL LAB



Swiss Light Source at PSI



# Practical considerations – Finding a SAXS beamline

Steps are generally the same:

- Find a beamline
- Talk with a beamline scientist
- Register and submit a proposal
- If you are going to do the experiment yourself
  - Safety training etc.

The screenshot shows the website <https://www.aps.anl.gov/Users-Information/About-Proposals/Apply-for-Time>. The navigation bar includes: About, Safety, Organization, APS User Info, APS-U, Machine Status, Beamlines, Media, and Search. Below the navigation bar are links for Long-Range Schedule, Industry, APS Highlights Book, APS Brochure, Science Highlights, Publications, APS/User News, Useful Links, and Directory. The main header reads "Advanced Photon Source" and "An Office of Science National User Facility", with the Argonne National Laboratory logo. The page content is divided into three columns:

- All About Proposals**: Includes "Users Home", "Apply for Beam Time" (with sub-links for Deadlines, Proposal Types, Concepts, Definitions, and Help), and "My APS Portal" with a colorful geometric logo.
- Apply for Beam Time**: Features a "Next Proposal Deadline" section with the following information:
  - The proposal submission deadline for **Run 2019-3** is **Friday, July 5, 2019, at 11:59 p.m. (Chicago time)**.
  - Questions: write to [gu\\_program@aps.anl.gov](mailto:gu_program@aps.anl.gov) or call 630-252-9090.
  - Please note that Chrome is not supported for the on-line proposal system.A "Log in to Proposal System" button is located at the bottom of this section.
- APS Contact Information**: Lists contact details for:
  - General Inquiries**: [apsuser@anl.gov](mailto:apsuser@anl.gov), (630) 252-9090, 8:00 am - 5:00 pm, Monday-Friday.
  - Floor Coordinator on Duty**: 630-252-0101 (on-call pager), From on-site phone: 2-0101.
  - Main Control Room**: 630-252-9424.
  - Safety Manager**: Paul Rossi

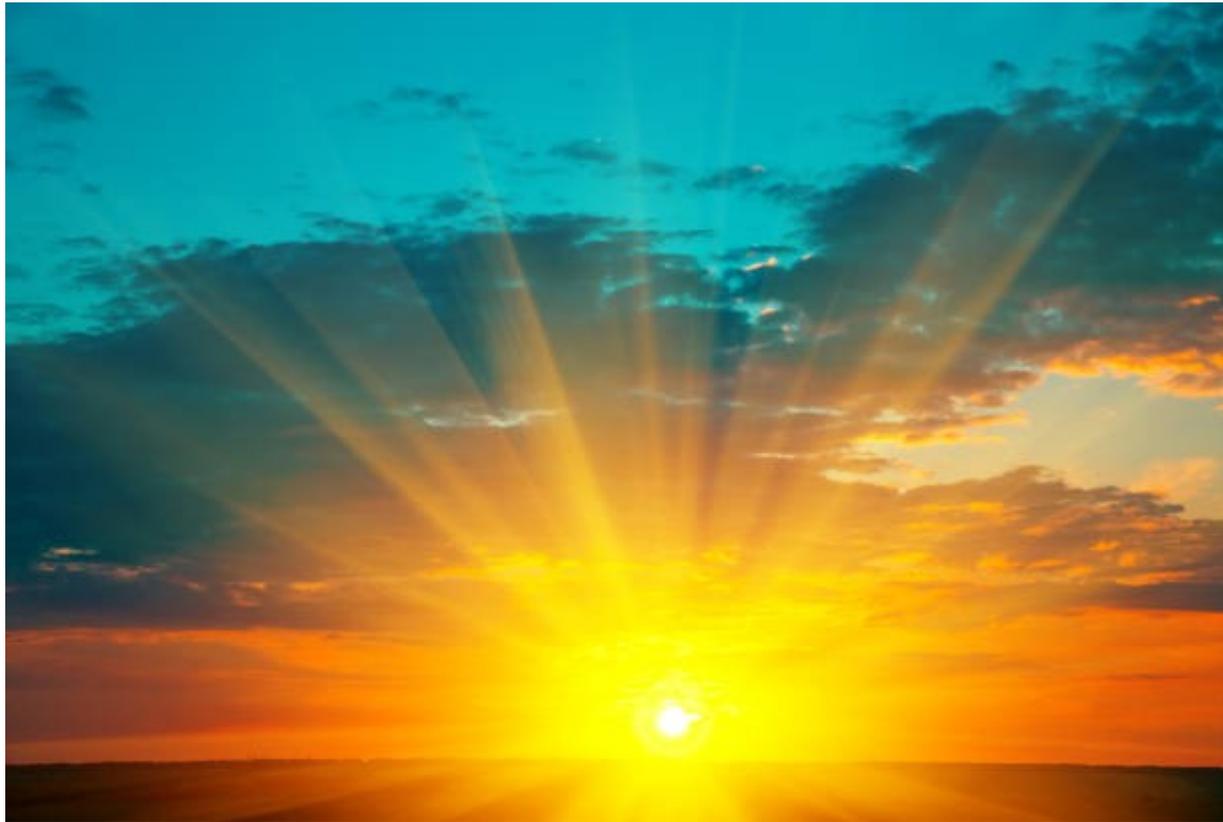
# Practical Considerations

If you are awarded time

- Bring a **TEAM!**
- Bring extra samples (ask colleagues).
- Expect to work every hour of your allocation!
  - e.g. if you have 2 days beamtime scheduled, expect to have someone working at the beamline 48 hours



APS at Argonne National Laboratory



# Practical considerations

## SANS beamlines

NCNR, NIST, Maryland  
HFIR, ORNL, Tennessee  
ISIS, RAL, UK  
ILL, Grenoble, France  
ANSTO, Sydney, Australia  
ESS, Lund, Sweden (2025)  
others...

NIST

Search NIST 

## NIST CENTER FOR NEUTRON RESEARCH

Logon to your  
NCNR-IMS  
account

Obtaining Beam  
Time

Arrange a visit to  
NCNR +

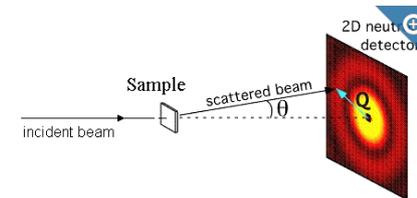
Planning Your  
Experiment +

Live Data

About NCNR +

Neutron  
Instruments  


## Small Angle Neutron Scattering (SANS)



### SANS Instruments

- [CHRNS VSANS](#)
- [CHRNS 30m SANS](#)
- [CHRNS USANS](#)
- [nSoft 10m SANS](#)
- [NG7 30m SANS](#)

Small-Angle Neutron Scattering (SANS) probes material structure on the nanometer ( $10^{-9}$  m) to micrometer ( $10^{-6}$  m) scale. Structures on this length scale are critical to the performance of advanced engineering materials.

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Science and Discovery

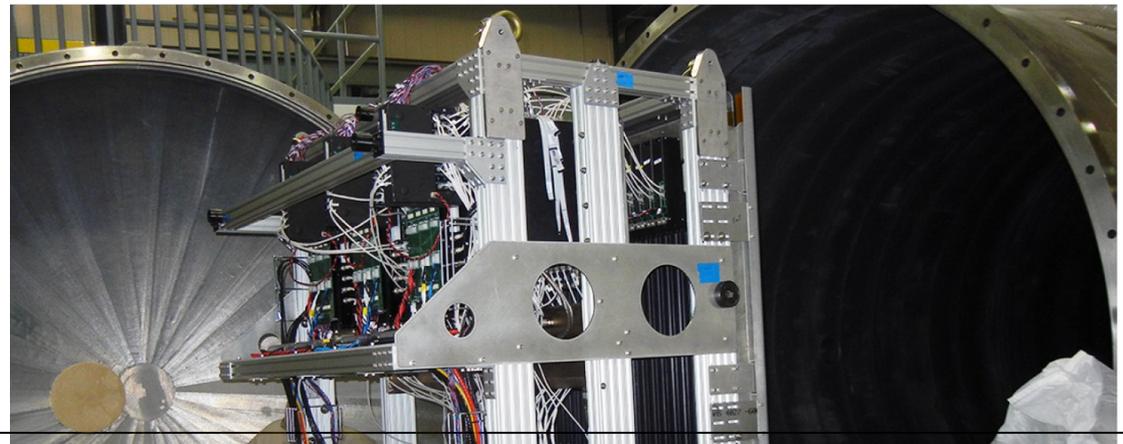
Neutron Sciences

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## Biological Small-Angle Neutron Scattering Instrument BIO-SANS | CG-3 | HFIR

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# SAXS vs SANS

	<u>SAXS</u>	SANS
<b>Features</b>	<u>msec</u> resolution for time-resolved measurements	D labeling and H/D contrast variation
	Superior q-resolution	Magnetic scattering
	Anomalous scattering ( <u>ASAXS</u> )	Conducive to extreme environments
	Small sample size	Nondestructive
<b>Complications</b>	Radiation damage to some samples	Incoherent scattering
	Parasitic scattering	H/D isotope effects
	Fluorescence	
	Beam stability	

# ASAXS

## Anomalous SAXS:

Allows limited contrast variation when the adsorption edge of one of the constituent elements is at an accessible energy range.

## Theory pioneered by Heinrich B. Stuhrmann:

*Q. Rev. Biophys.* 14, 433 (1981)

*Adv. Polym. Sci.* 67, 123 (1985)

Stuhrmann analyzed metal containing proteins such as hemoglobin, ferritin, and the anomalous effect on the radius of gyration of DNA near the absorption edge of counterions.

In the case of the large subunit of ribosome (1500 kD), measurements near phosphorous K-edge allowed separation of all three partial intensities. *Stuhrmann. J. Appl. Cryst.* 2007. 40:s23



*Heinrich B Stuhrmann  
Guinier prize 2006*

# ASAXS

$$f(\lambda) = f_0 + f'(\lambda) + \mathbf{i}f''(\lambda)$$

$$|f| = [(f_0 + f')^2 + f''^2]^{\frac{1}{2}}$$

Stuhrmann 1981:  $f''$  via absorption vs wavelength for bound iron.

$f'$  via  $f''$  using the Kramers-Kronig relation...

Tabulated values are available for most elements.

Corrected  $I(q)$  curves were produced, compared.

Multipole expansion for scattering density

→ distance distributions for iron were estimated.

KK:

Re/Im of Fourier

1-1 Even odd

Generalized in *V. J. Pinfield and D.J. Scott. PLoS ONE. 2014 9(4): e95664*

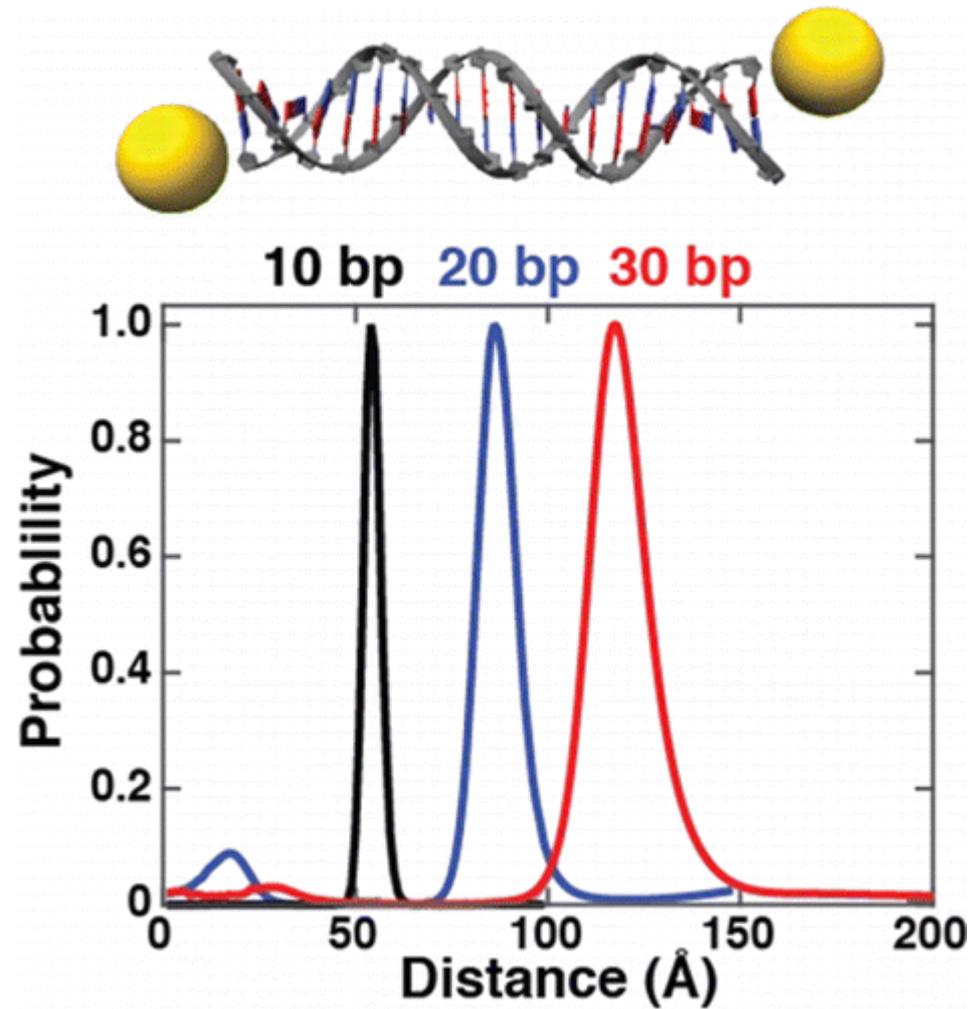
**Table 2.** Distances between label atoms or nanocrystals.

Molecule	Actual distance between labels/Å	Calculated distance between labels/Å
10 bp DNA, atom labels	37.3	----
10 bp DNA, nanocrystal	50.5	51
20 bp DNA, nanocrystal	60.7	61
50 bp DNA, nanocrystal	142.0	143
100 bp DNA, nanocrystal	269.6	270
200 bp DNA, nanocrystal	672.0	673

*The distance between the label atoms or nanocrystals, as defined in the coordinate files, and determined by the anomalous SAXS simulation.*

doi:10.1371/journal.pone.0095664.t002

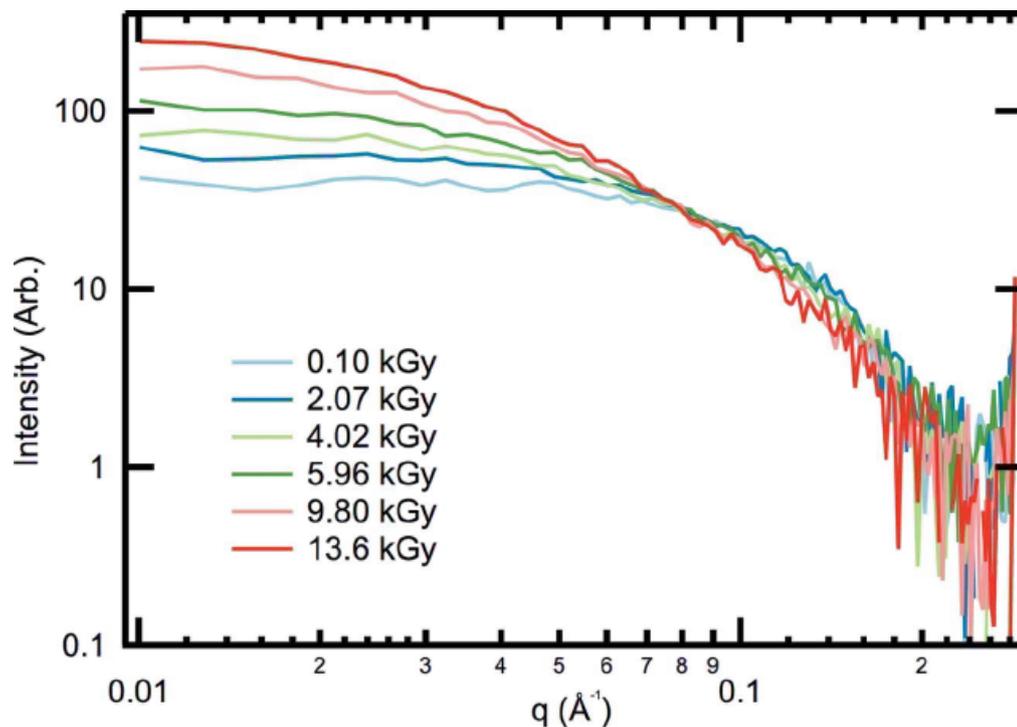
# ASAXS



APS  
ID-12

# SAXS – Radiation damage

X-ray-induced radiation damage can cause macro- molecule aggregation, fragmentation, conformation changes and unfolding, all of which can be detected by SAXS. Radiation damage is therefore a major obstacle for SAXS, and descriptions of dedicated biological SAXS beamlines acknowledge the need to check for and avoid radiation damage.



Radiation damage in most contexts is a function of Dose ( $\text{Gy} = \text{J kg}^{-1}$ ).

$$\text{Dose} = \frac{ftAE_{\gamma}}{\rho l}$$

- $f$  – flux density
- $t$  – exposure time
- $A$  – fraction of incident energy absorbed
- $E_{\gamma}$  – energy of photon
- $\rho$  – sample density
- $l$  – path length

# SAXS – Radiation damage

Minimize:

Reduce exposure time

$$\text{Dose} = \frac{ftAE_{\gamma}}{\rho l}$$

Decrease volume irradiated

Oscillating or continuous flow

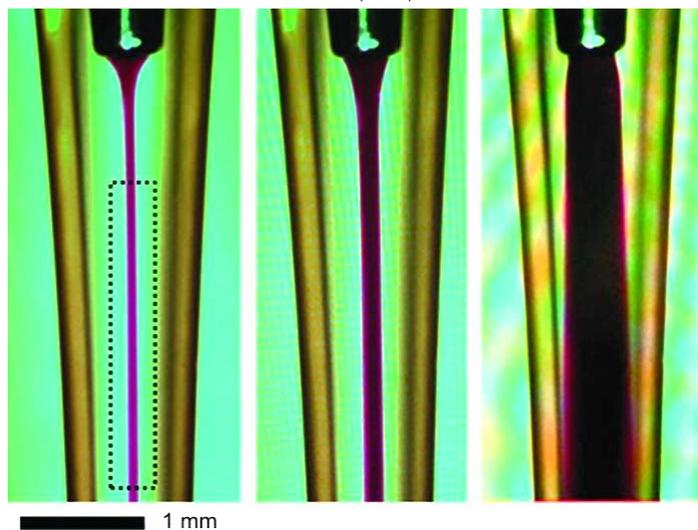
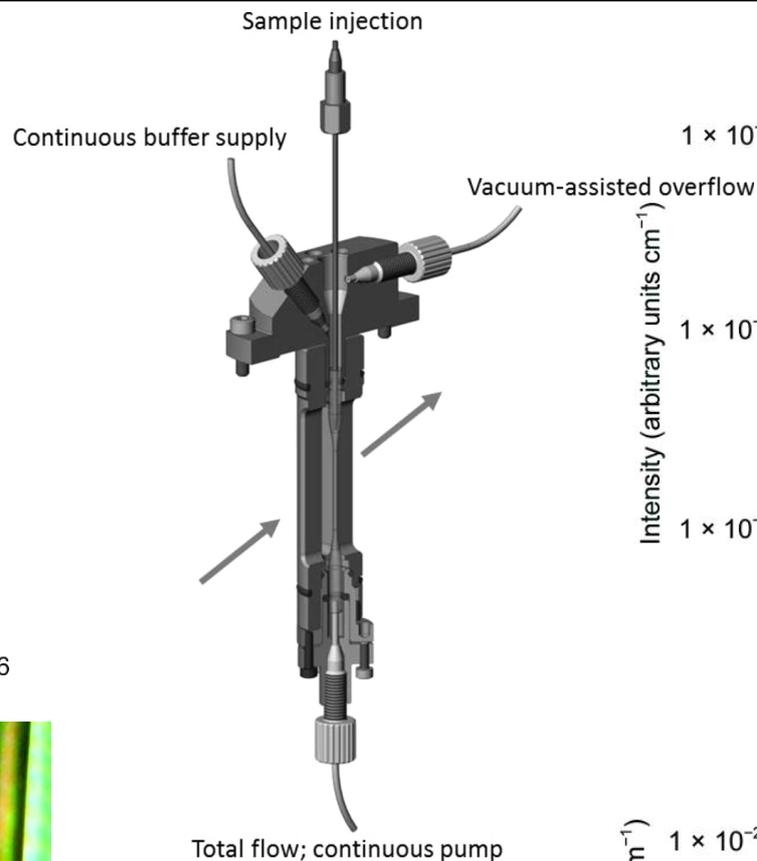
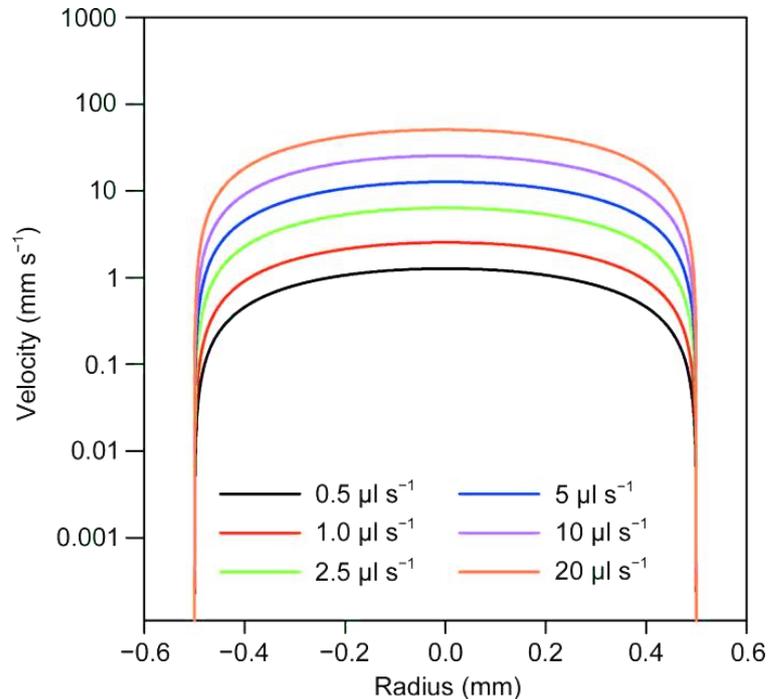
Defocusing the beam

Buffer additives to competitively bind with free radicals or by inhibit aggregation

Glycerol

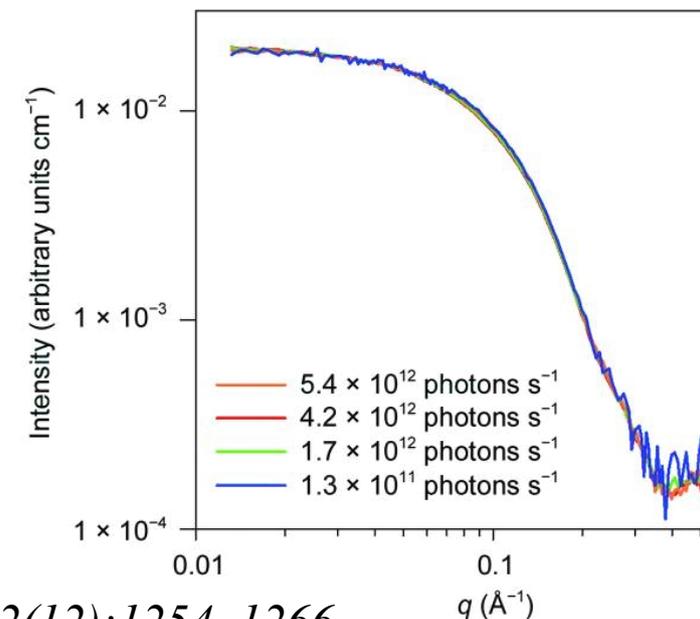
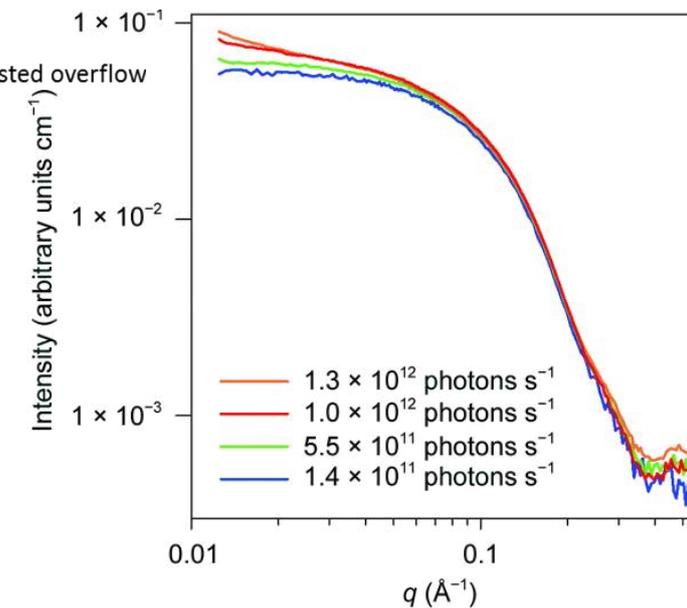
Cryo-SAXS

# SAXS - Coflow

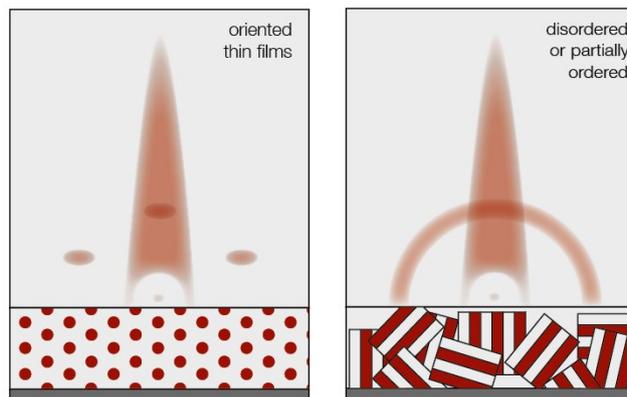
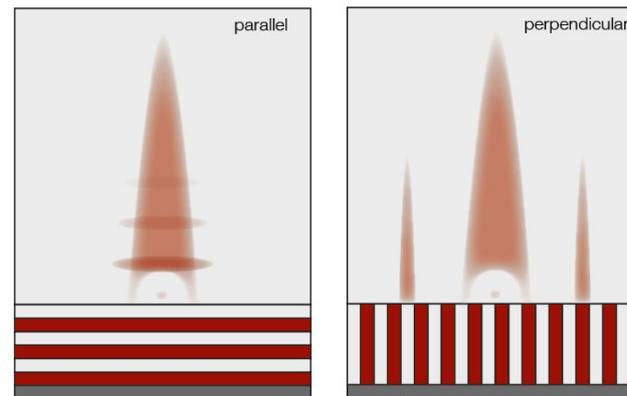
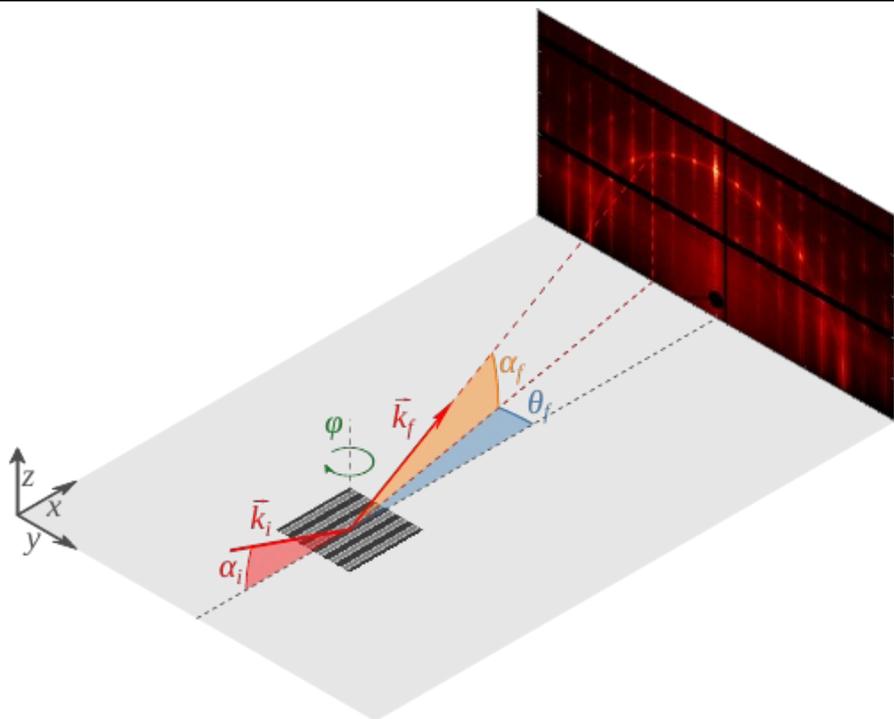


ANSTO / SAXS/WAXS  
APS / BioCAT  
SSRL / BL4.2  
SOLEIL / SWING  
MAX IV / CoSAXS

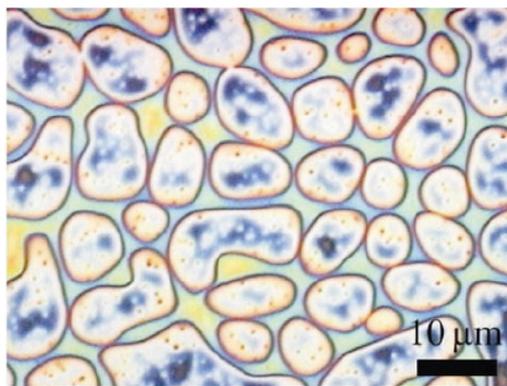
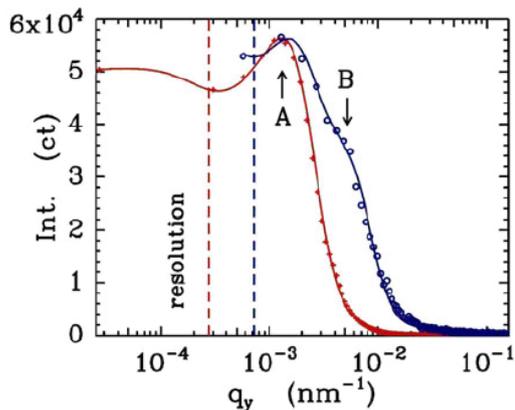
## RNase A



# Surface studies - GISAS



Müller-Buschbaum P. (2009) *A Basic Introduction to Grazing Incidence Small-Angle X-Ray Scattering. Lecture Notes in Physics, vol 776, Springer*



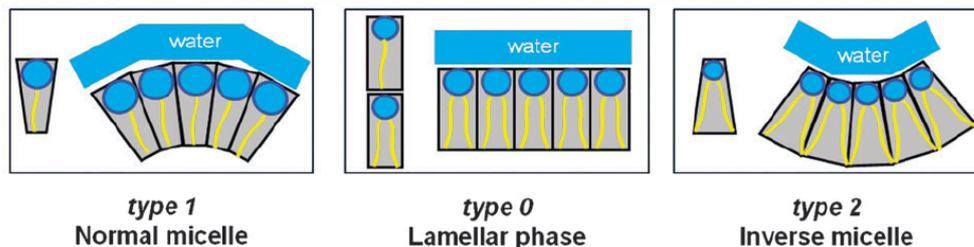
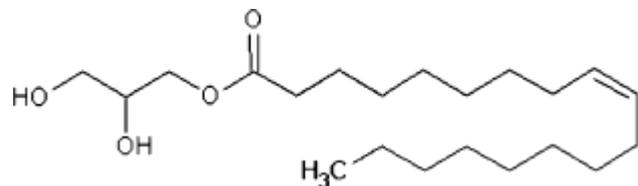
<https://wiki.anton-paar.com/en/grazing-incidence-small-angle-x-ray-scattering-gisaxs>

Blend films of PS and PnBA  
 "A" most prominent in-plane length

Müller-Buschbaum P., *Prog. in Colloid & Polymer Sci.* 2006 doi:10.1007/2882\_031

# Liquid crystal SAXS

Liquid crystal structures  
e.g. Monoolein thermotropic and lyotropic



Critical Packing Parameter  
(Shape Factor)

$$\gamma = v/a_0 l_c$$

Head group  
(hydrophilic)

Alkyl chain  
(hydrophobic)

$v$  = molecular volume

$l_c$  = effective maximum  
chain extension

$a_0$  = optimum headgroup area



Spherical

Cylindrical

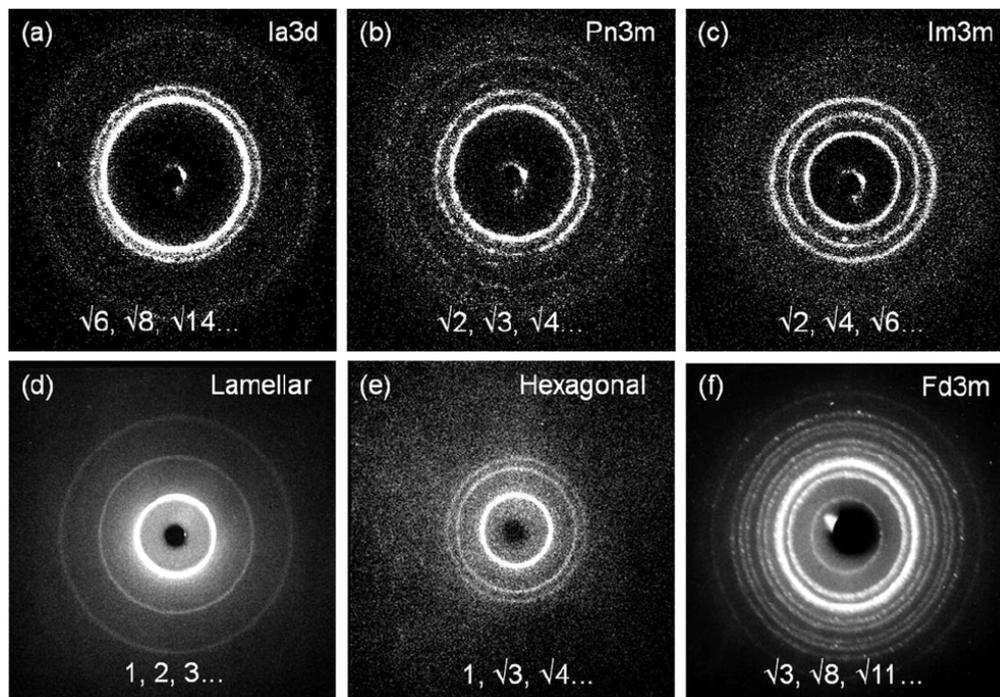
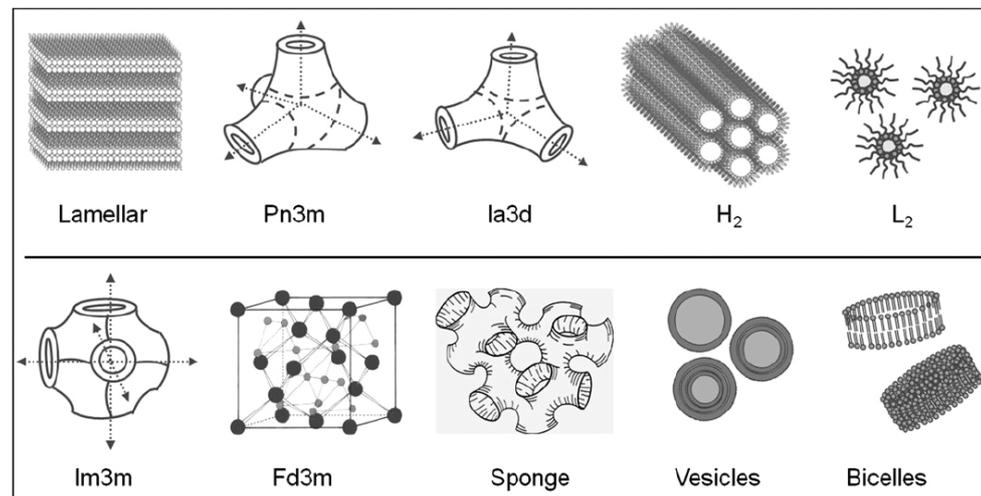
Lamellar

Inverse (spherical, cubic etc...)



**type 1**

**type 2**



# Monodispersity revisited

*Svergun* – 2003

Shannon channels =  $D_{max} \cdot q\text{-range} / \pi$

“the number of [obtainable parameters] typically does not exceed **10–15**”

*Hub* – 2018

“... generally accepted that experimental SWAXS curves do not contain more than 10–30 independent data points.”

Monodispersity → maximize information content / species

Even if you purify immediately before SAXS measurements and inject each fraction or a pool of fractions, you still have a chance that the sample will either aggregate or degrade during operations

# SEC-SAXS

- High pressure liquid chromatography or FPLC (Fast protein liquid chromatography) on line with the SAXS cell
- Individual peaks are more likely to be monodisperse
- First use paper, available to users who could self-manage FPLC
  - *Mathew, E., Mirza, A., & Menhart, N. (2004). Liquid-chromatography-coupled SAXS for accurate sizing of aggregating proteins. J. Synchrotron Rad. 11, 314-318.*
- First setup with user HPLC support
  - *David, G. & Pérez, J. (2009). Combined sampler robot and high-performance liquid chromatography: a fully automated system for biological small-angle X-ray scattering experiments at the Synchrotron SOLEIL SWING beamline. J. Appl. Cryst. 42, 892-900*
- Implementations (not guaranteed exhaustive)
  - ID-18 BioSAXS/APS
  - BL4.2/SSRL
  - CHESS/MacCHESS
  - SWING/SOLEIL
  - BM-29/ESRF
  - I22/Diamond
  - P12/Petra
  - SR13 ID01/Australian Synchrotron

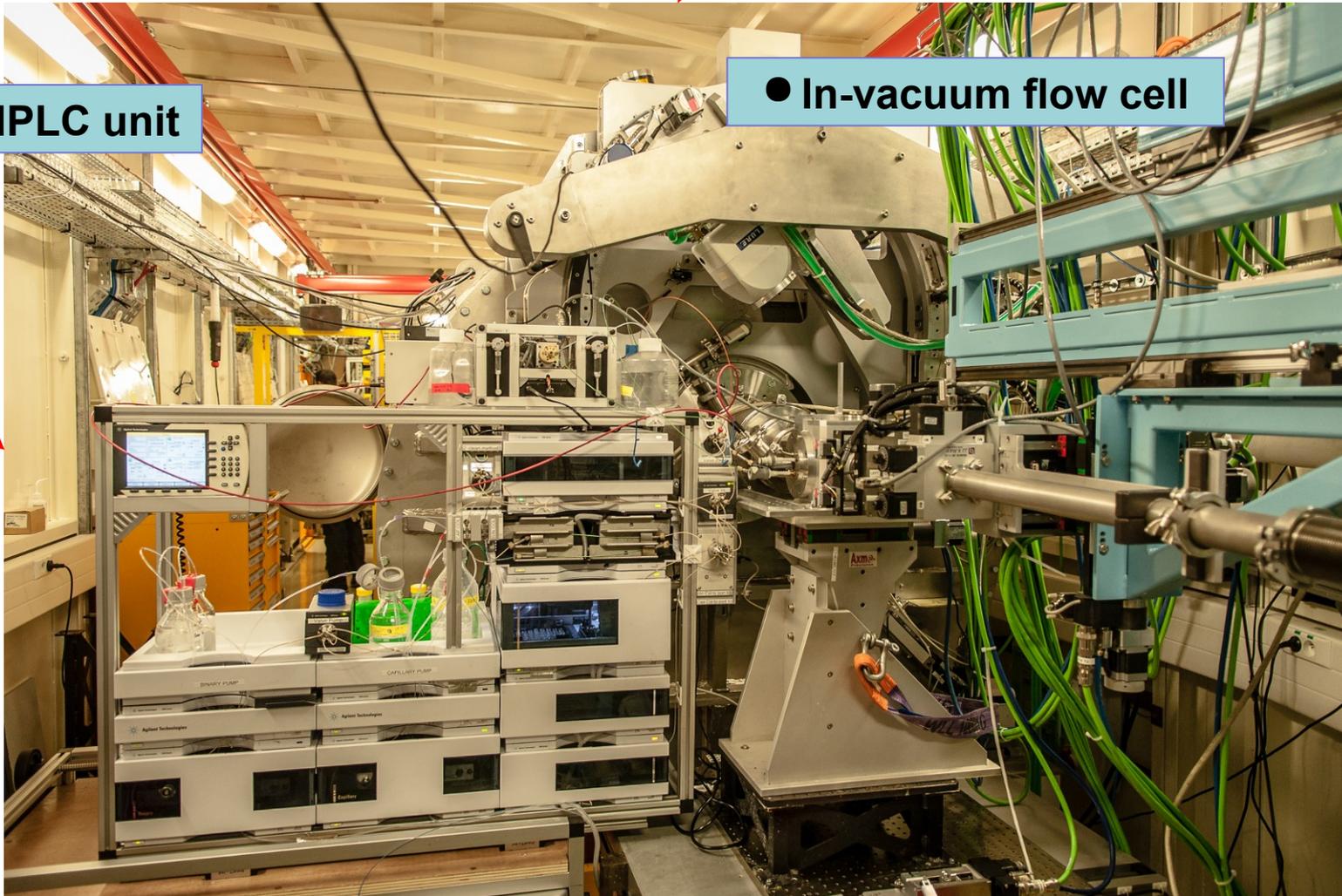
# SEC-SAXS

- Inline HPLC/MALS system

SWING/SOLEIL

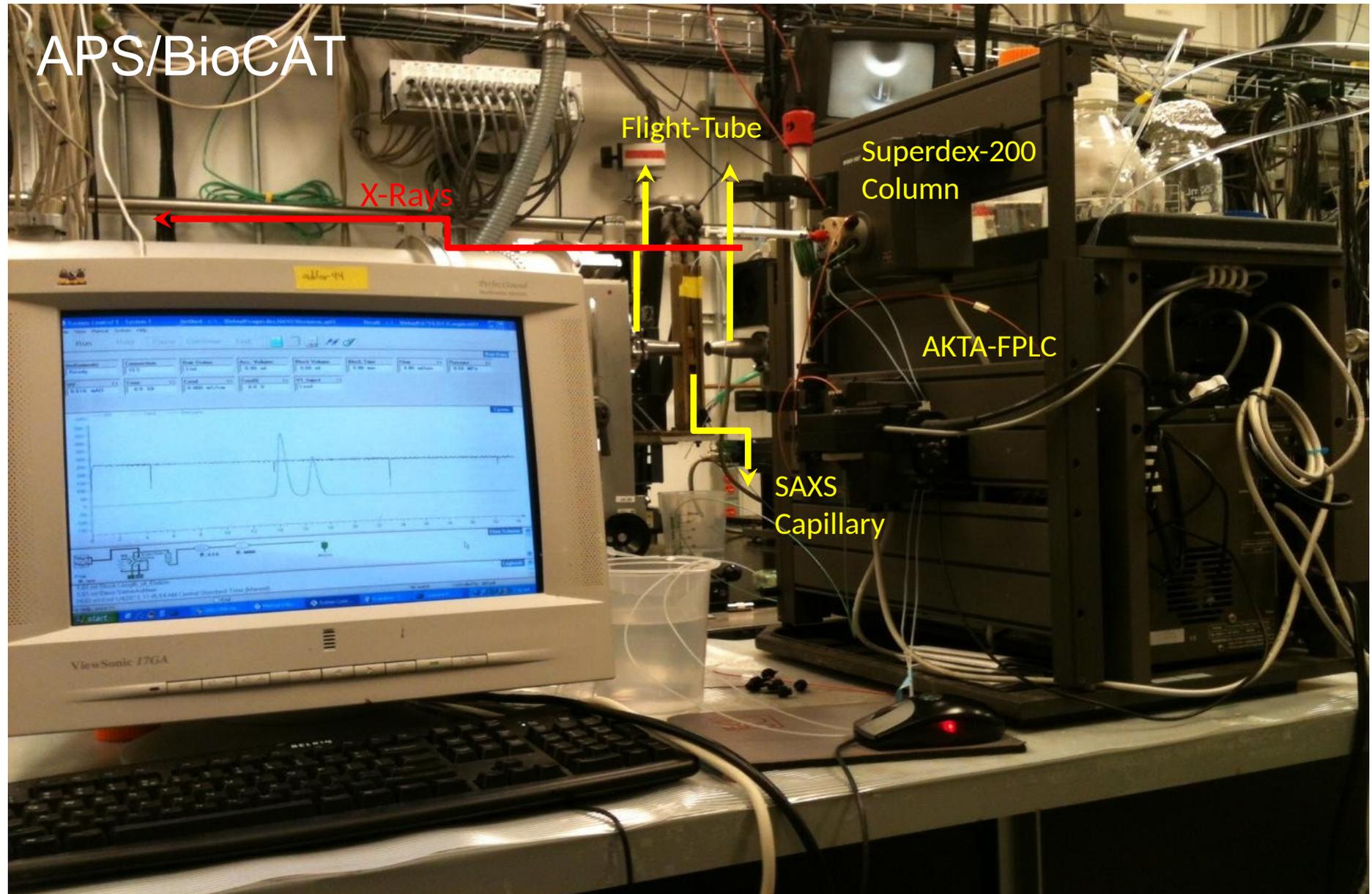
- HPLC unit

- In-vacuum flow cell



*Photo credit: Javier Perez*

# SEC-SAXS

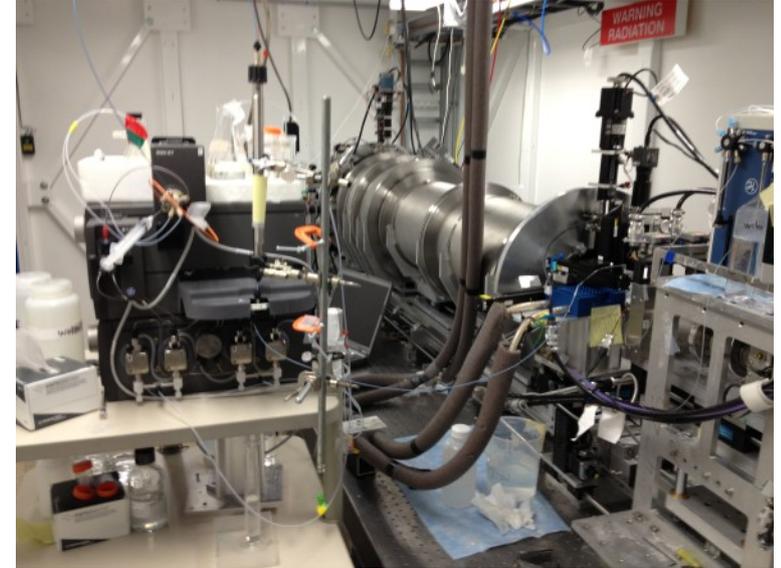


Slide Credit: Srinivas Chakravarthy

# SEC-SAXS

## SSRL/BL4.2

- The online FPLC-SAXS system at the BioSAXS beamline BL4-2
- consists of an Akta Ettan with low volume (2.5ml) SEC columns:
  - Superdex 200
  - Superose 6
  - Or bring your own
- The system uses the same flow path as the
- regular “autosampler” setup at the beamline:
  - rapid switch-in of the FPLC system during normal data collection
  - FPLC-SAXS and “autosampler” results can be compared quickly
- sample requirement:
  - typically 50ul of 5mg/ml sample
  - each run requires 3 ml of buffer and takes roughly an hour
- Automated data analysis scripts allow easy tracking of experimental results during experiment
- More information on our website:
  - <http://www-ssrl.slac.stanford.edu/~saxs/>

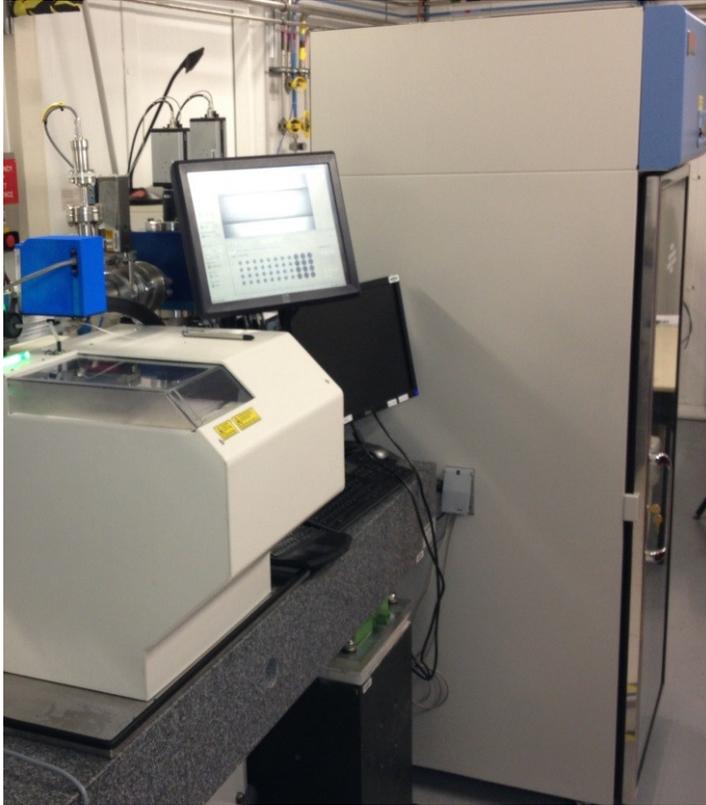


*Slide Credit: Thomas Weiss*



# SEC-SAXS

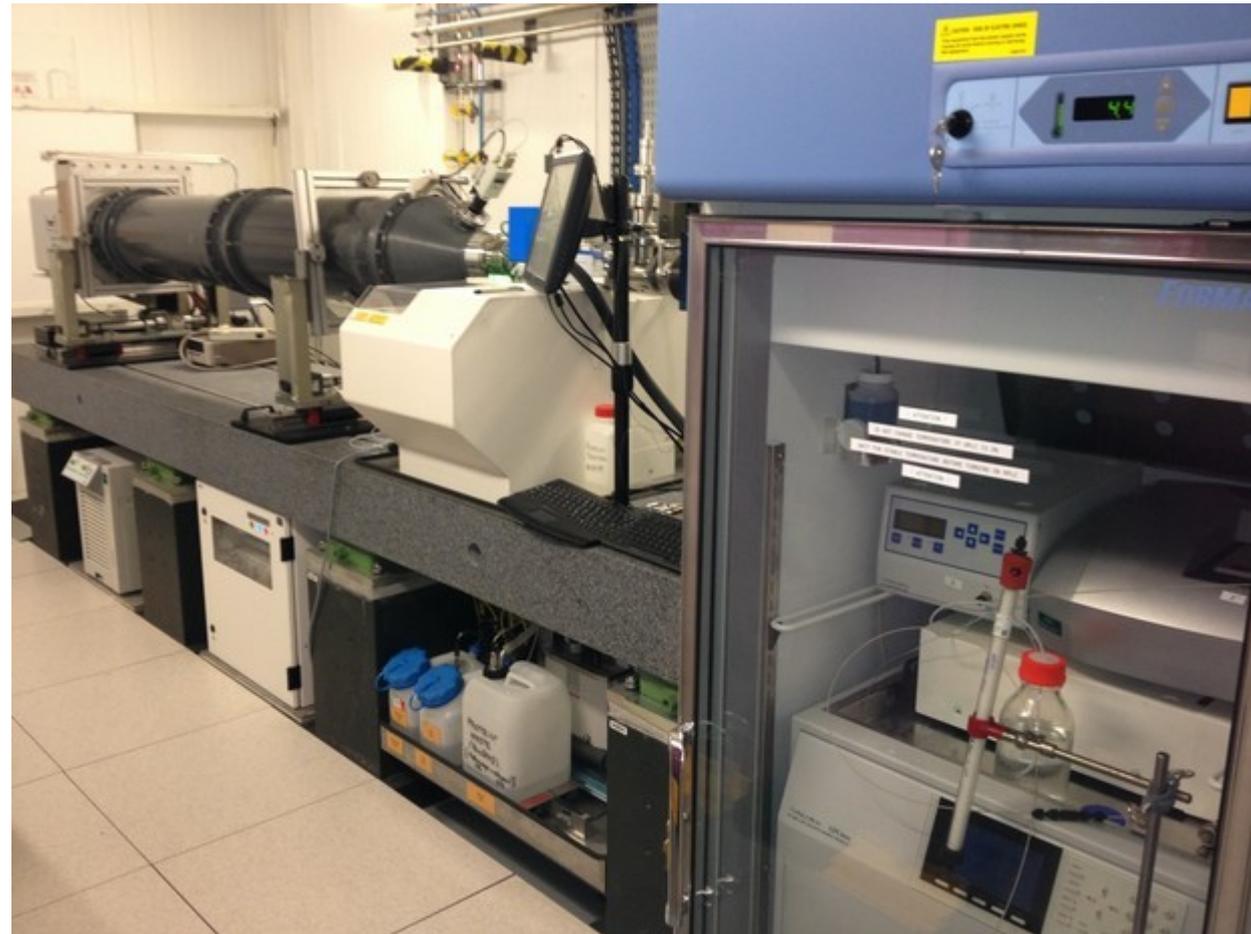
## BM29/ESRF



- Automated switching between SEC and SC for efficient use

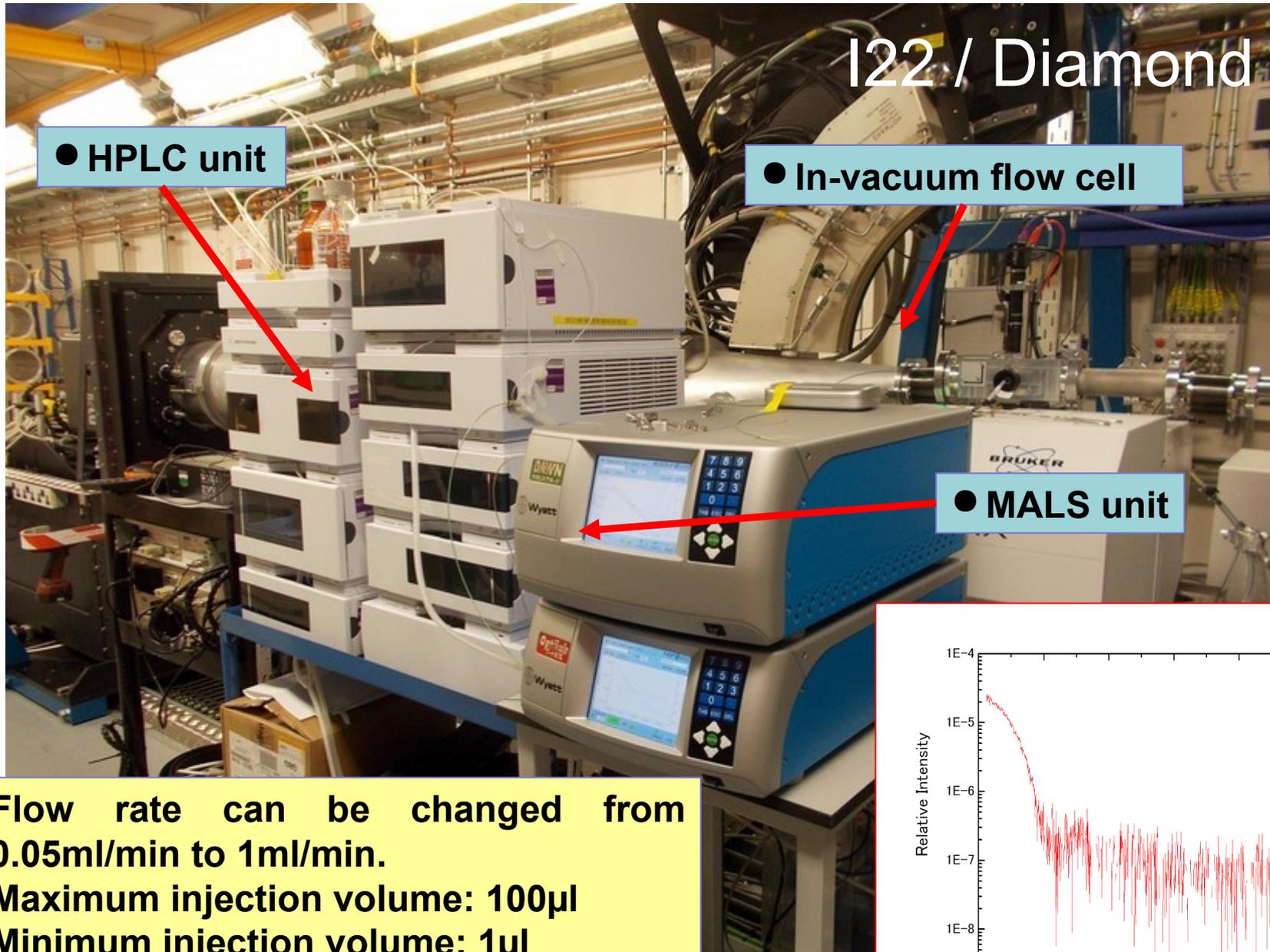


- Integrated Sample changer and Online-SEC
- SEC units housed in temperature controlled cabinet (4 -25 °C)



*Slide Credit: Adam Round*

# SEC-SAXS



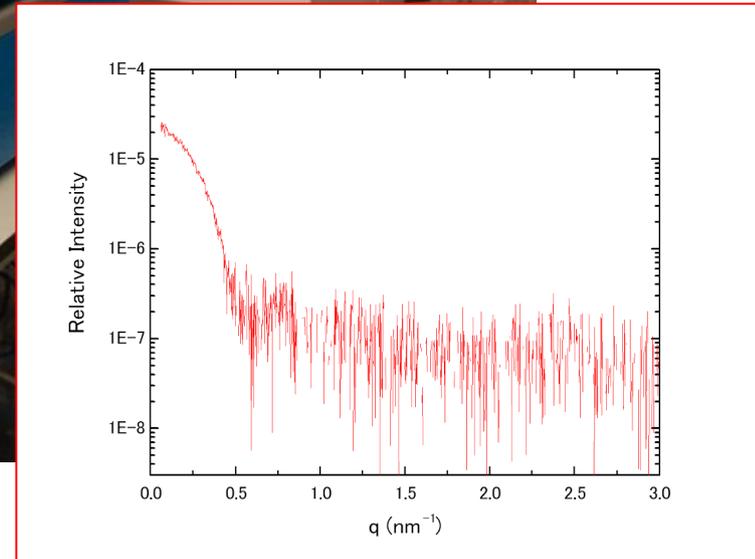
I22 / Diamond

● HPLC unit

● In-vacuum flow cell

● MALS unit

- Flow rate can be changed from 0.05ml/min to 1ml/min.
- Maximum injection volume: 100 $\mu$ l
- Minimum injection volume: 1 $\mu$ l



Slide Credit: Katsuaki Inoue

# SEC-SAXS

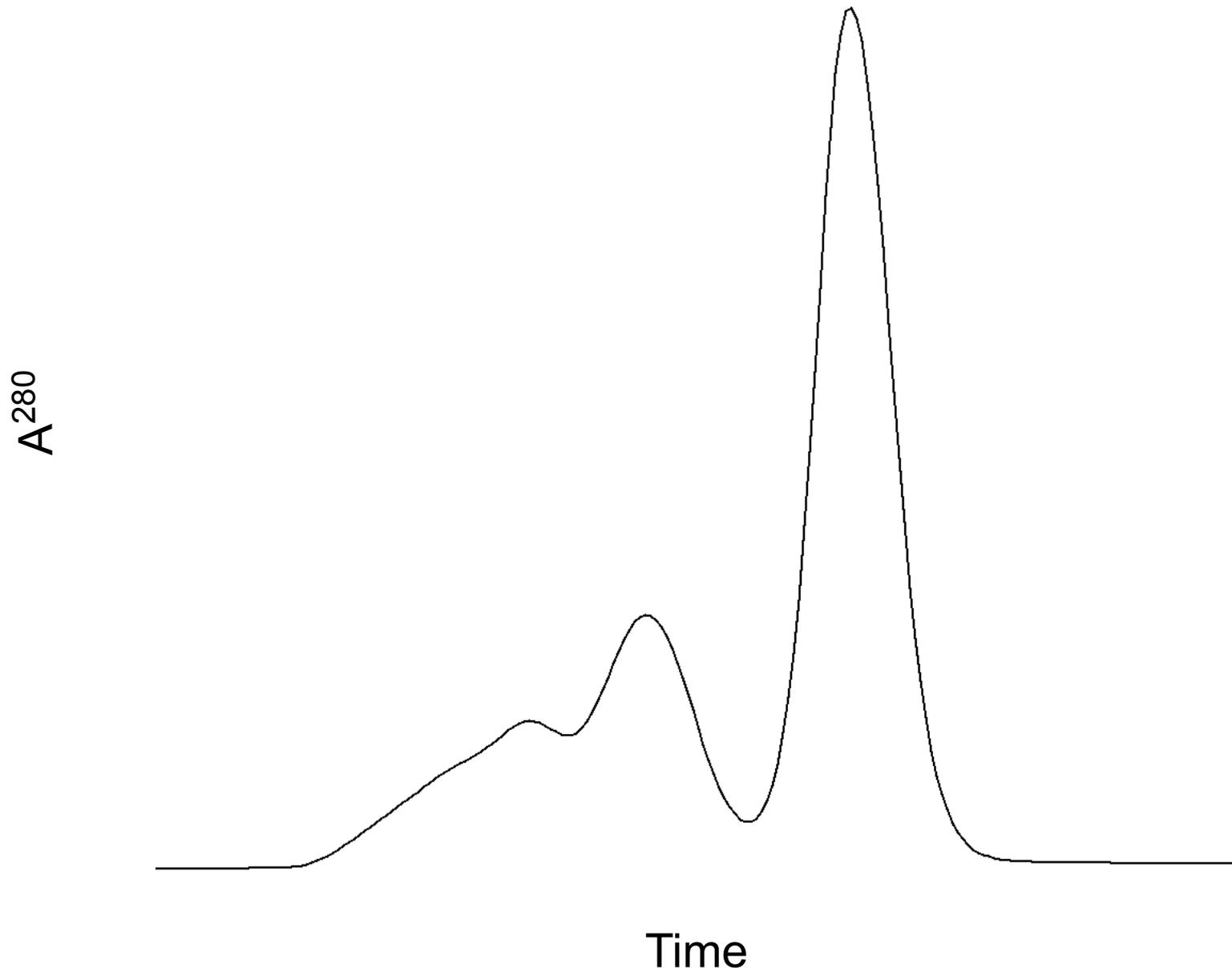
- Separate immediately before measuring
- Individual peaks are more likely to be monodisperse
- Now available as primary method of analysis at multiple beamlines
- Conclusion of ACA 2014 session 4.2.4 [ACA Reflexions Fall 2014]:

*“The consensus that emerged was that SEC-SAXS may become the standard data collection strategy for biological samples, as a large number of samples that were heretofore believed to be monodisperse have been shown to be polydisperse when analyzed with online SEC-SAXS setups.”*

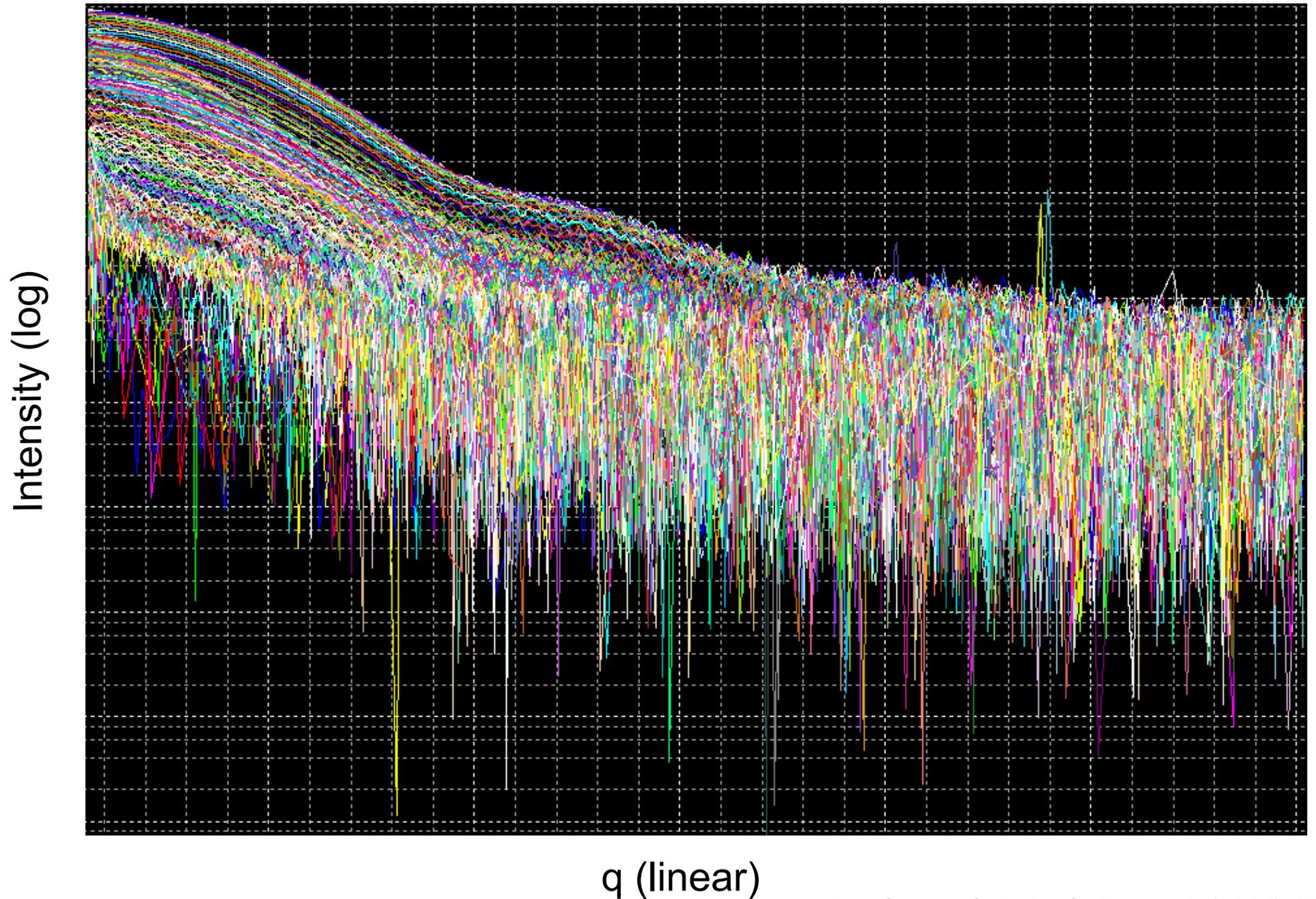


Nicolas Wolff, Sophie Zinn-Justin, Nigel Kirby, Alvin Acerbo, Srinivas Chakravarthy, Javier Pérez, Alexey Kikhney, Emre Brookes, Adam Round, David Lambright.

# UV Trace Aldolase

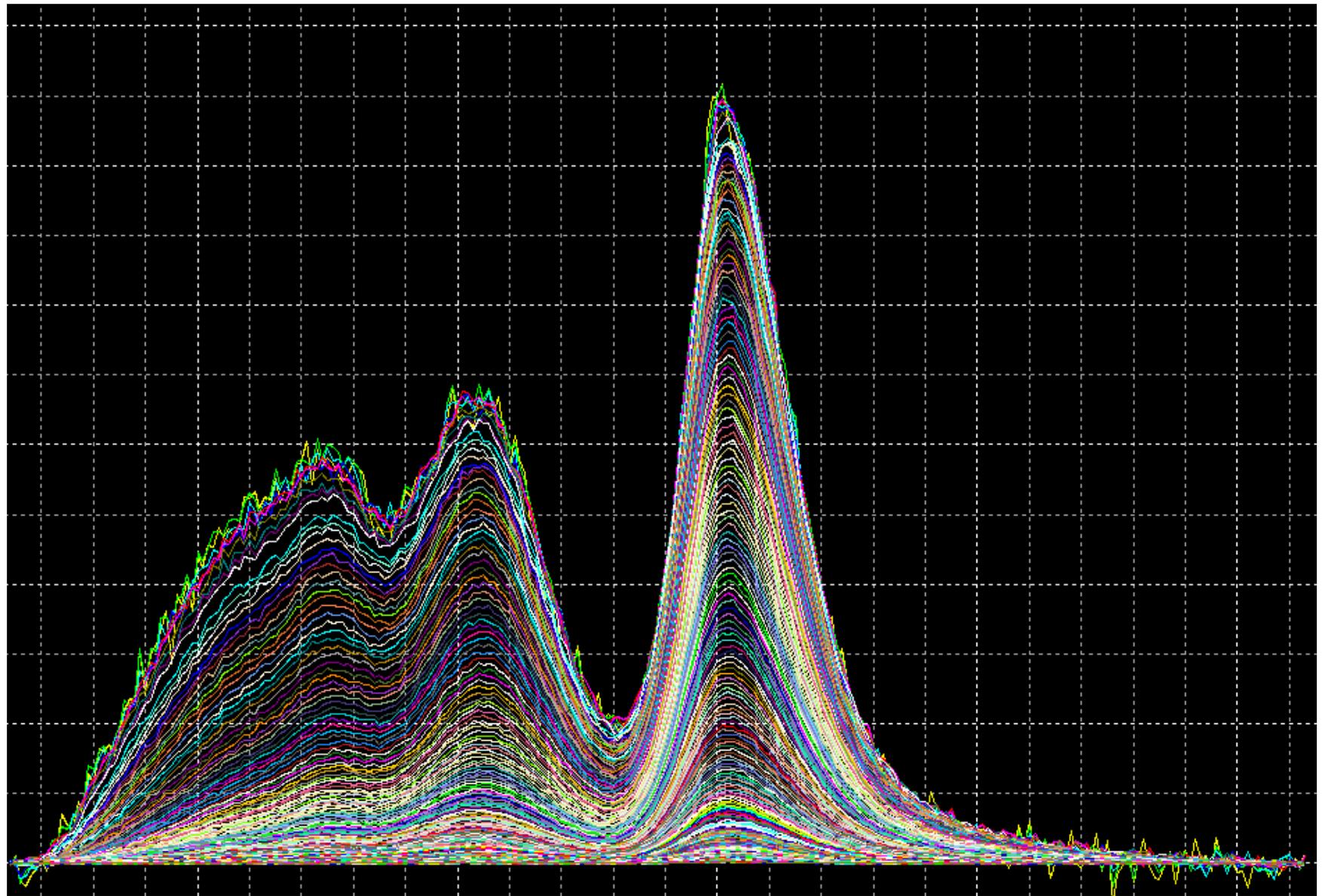


# SEC-SAXS $I(q)$ profiles Aldolase



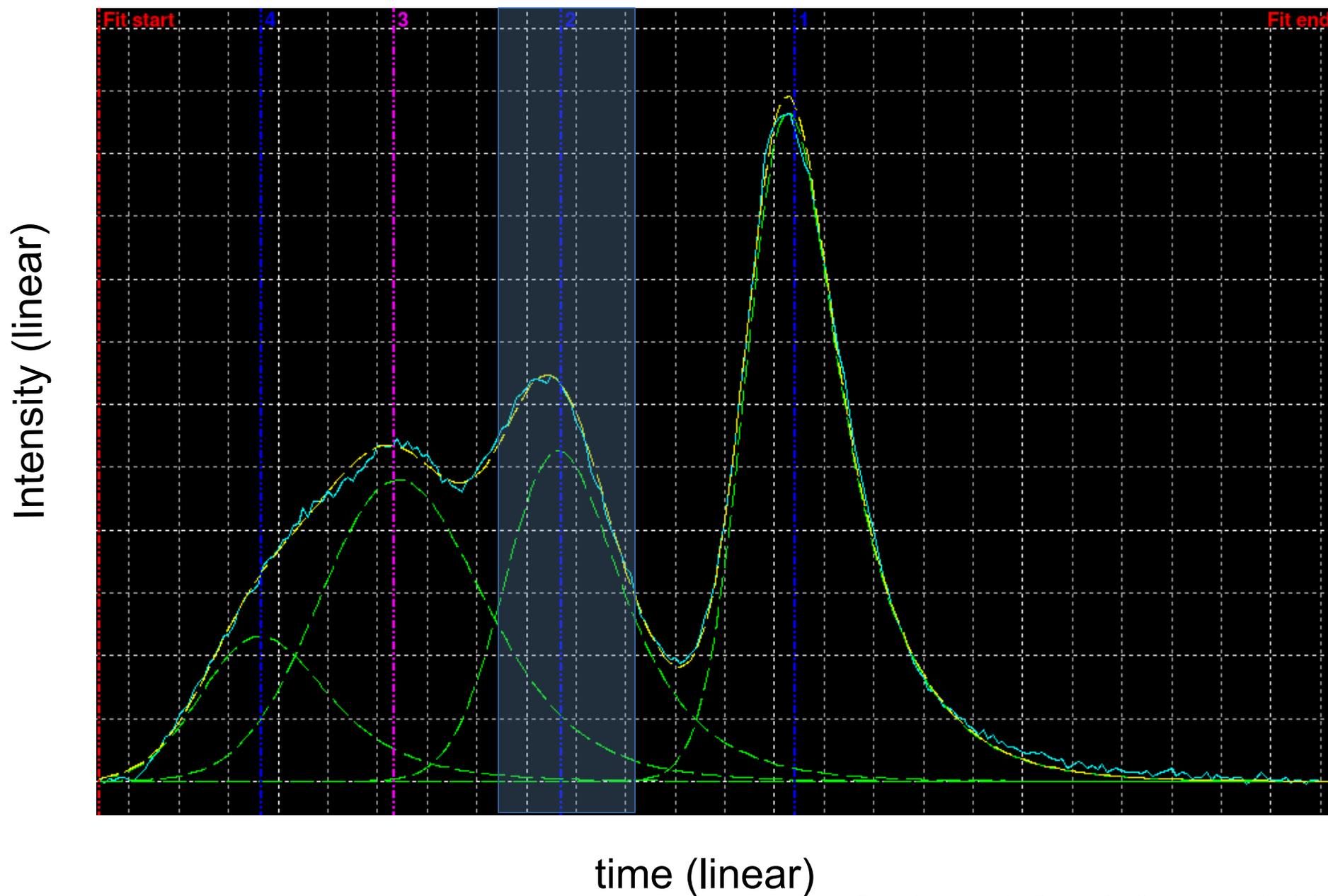
# SEC-SAXS $I(t)$ profiles Aldolase

Intensity (linear)



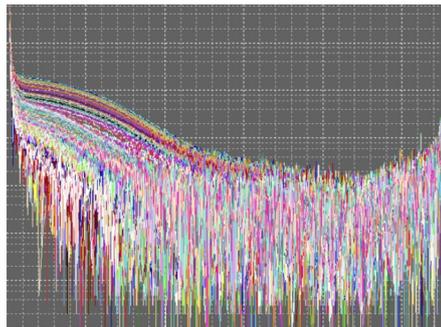
time (linear)

# Aldolase Gaussian fit of one $I(t)$

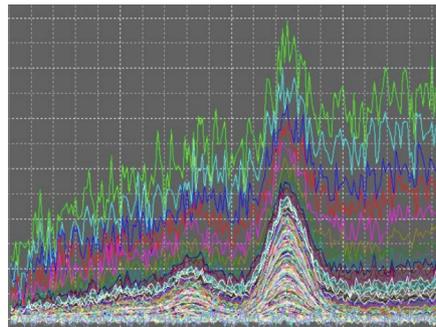


# SEC-SAXS deconvolution of peaks

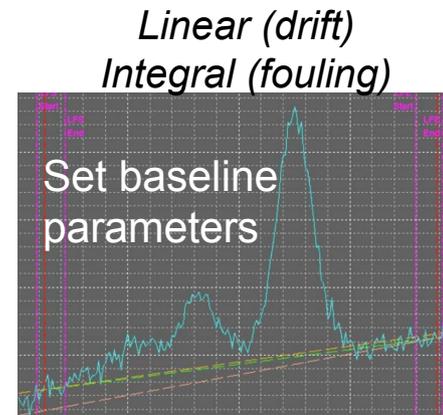
Collect SEC-SAXS data



Make  $I(t)$

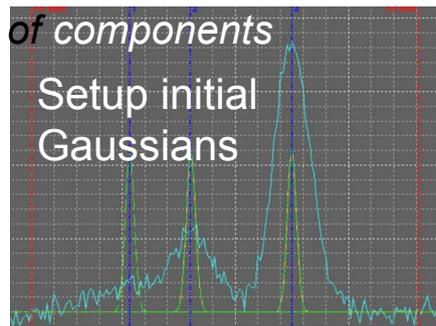


Select typical curve

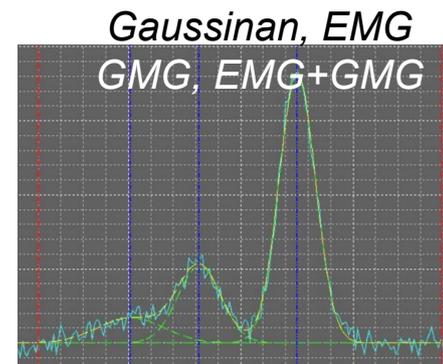


optional SVD to inform number of components

Select typical curve

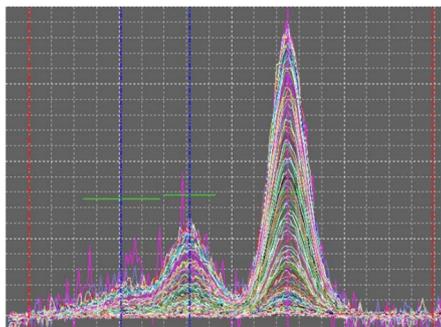


Gauss Fit



UV or RI, simultaneously fit for accurate MW computations

Global Gauss Fit



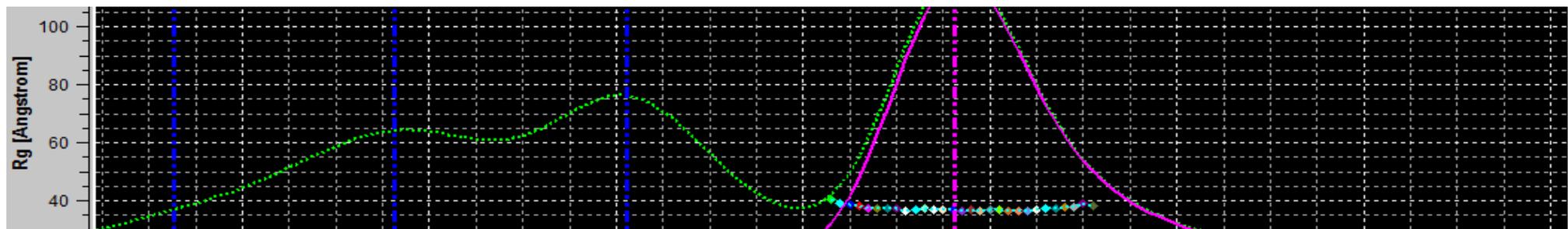
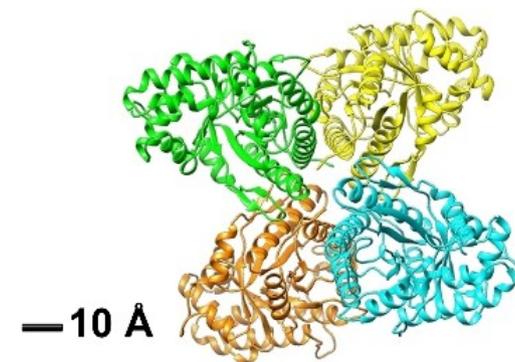
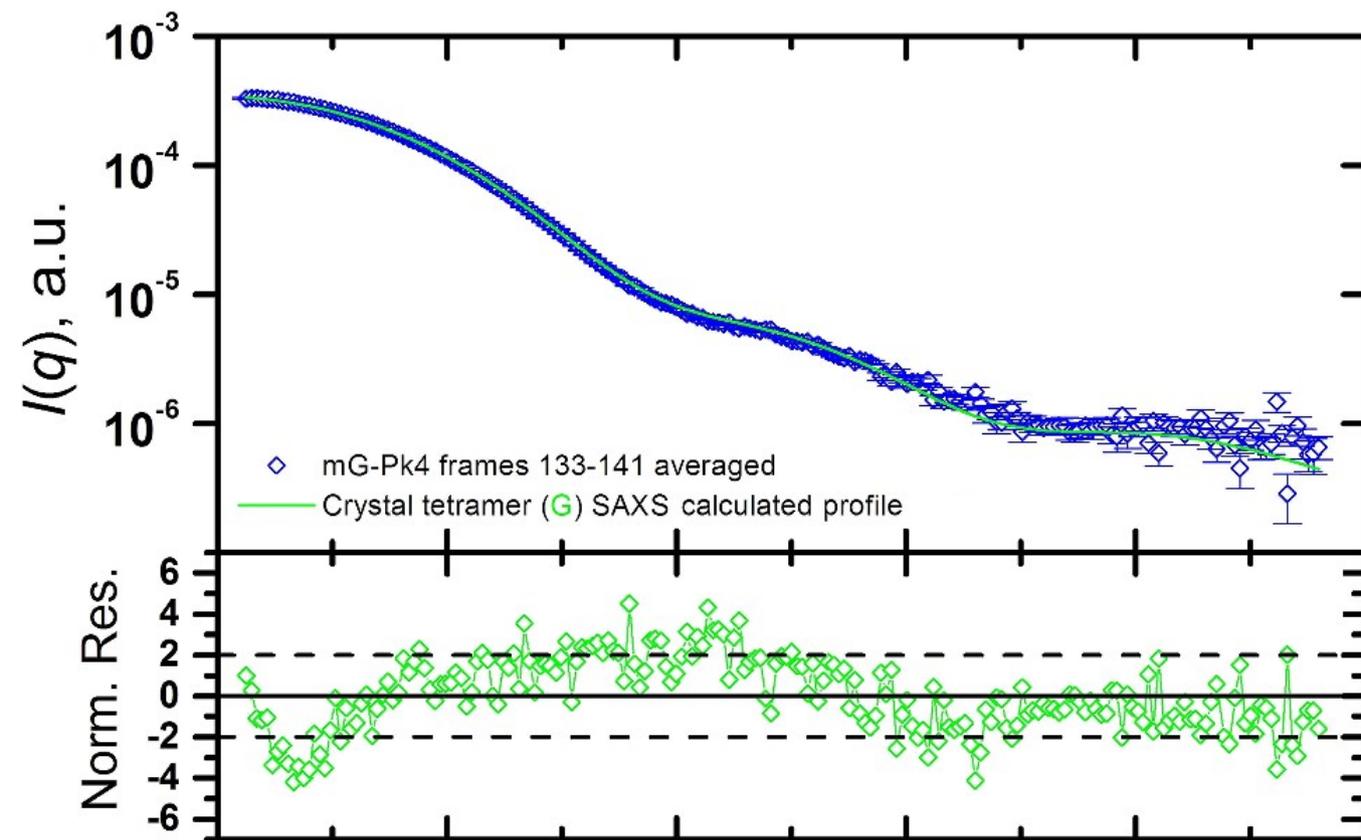
Opt. apply to conc. curve



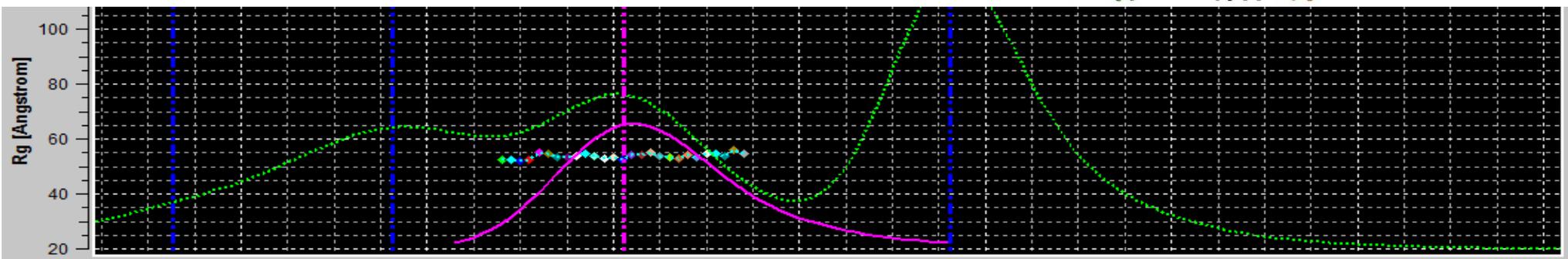
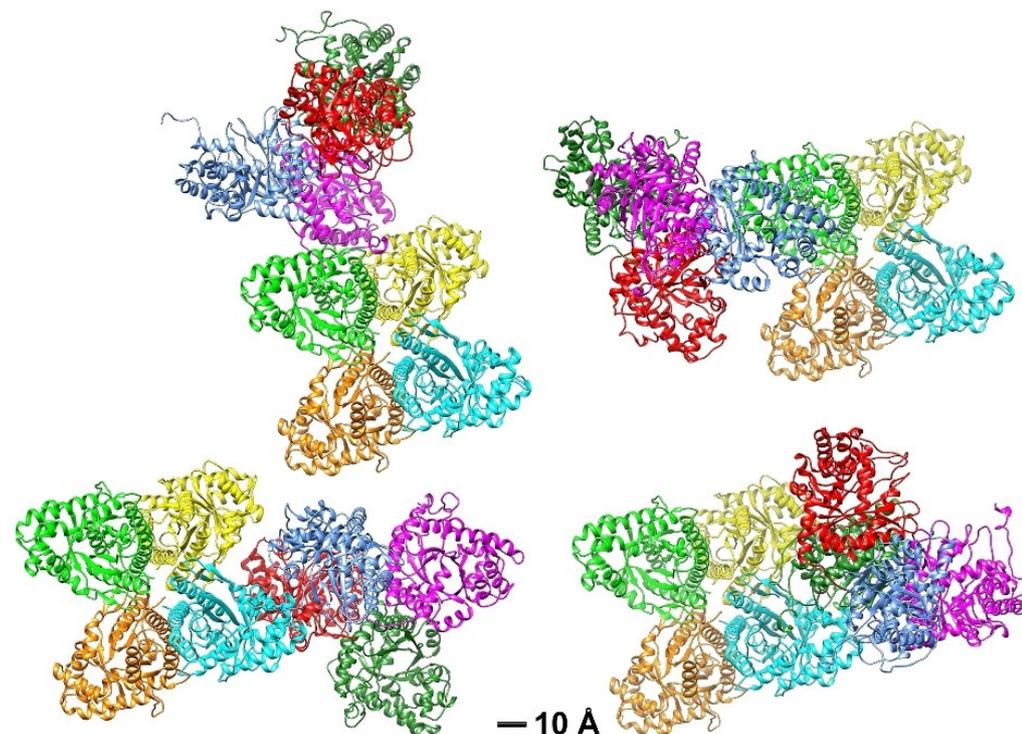
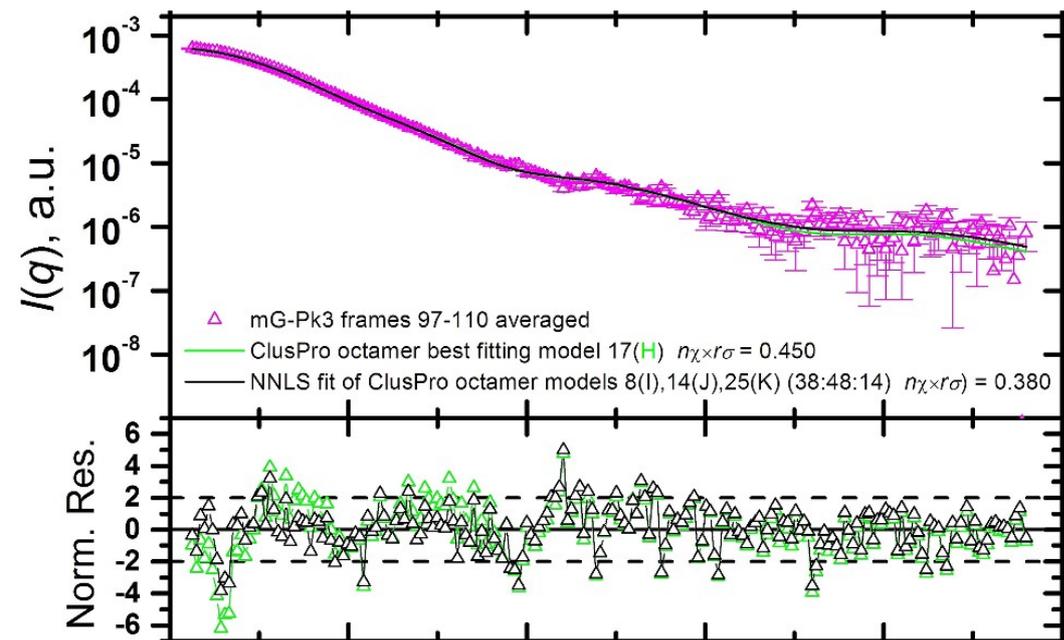
Make  $I(q)$

Set of  $I(q)$  curves for each Gaussian peak

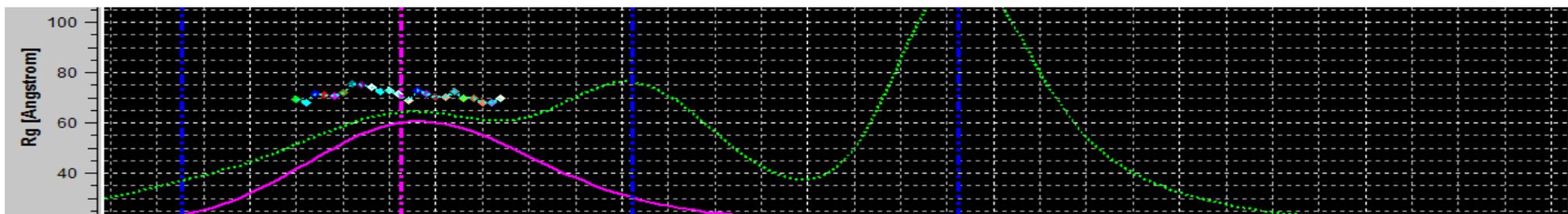
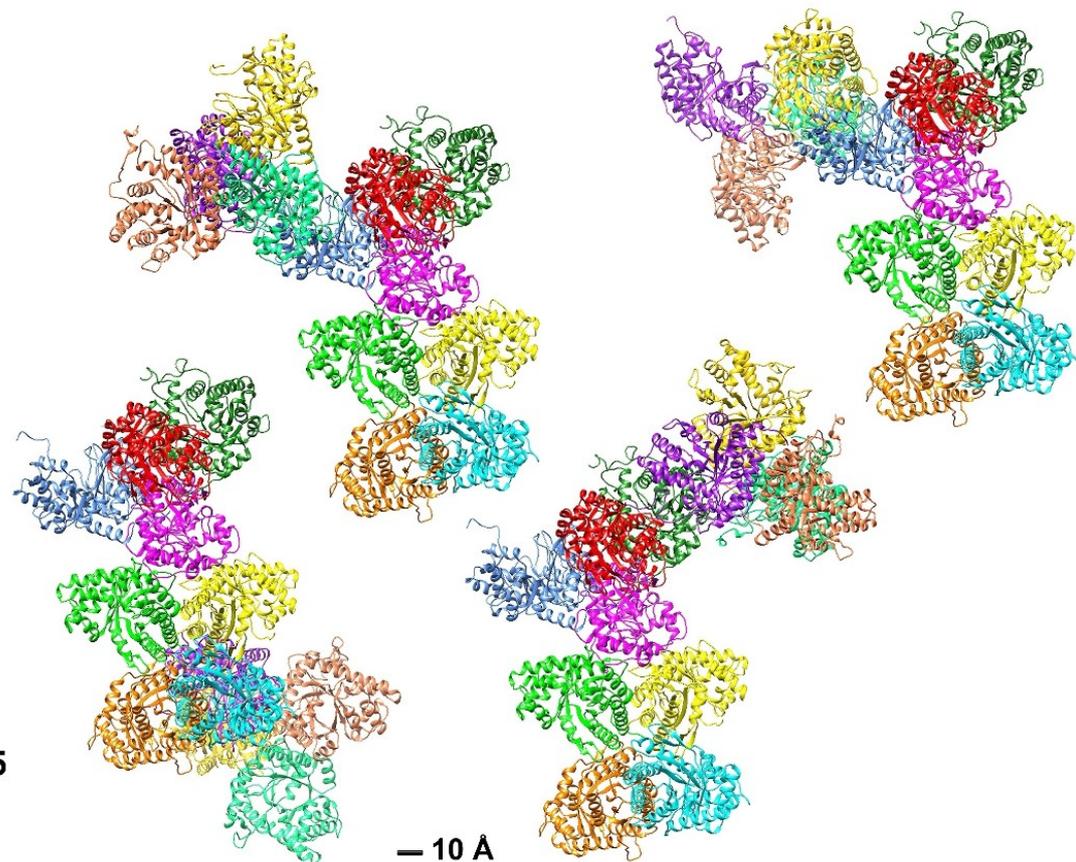
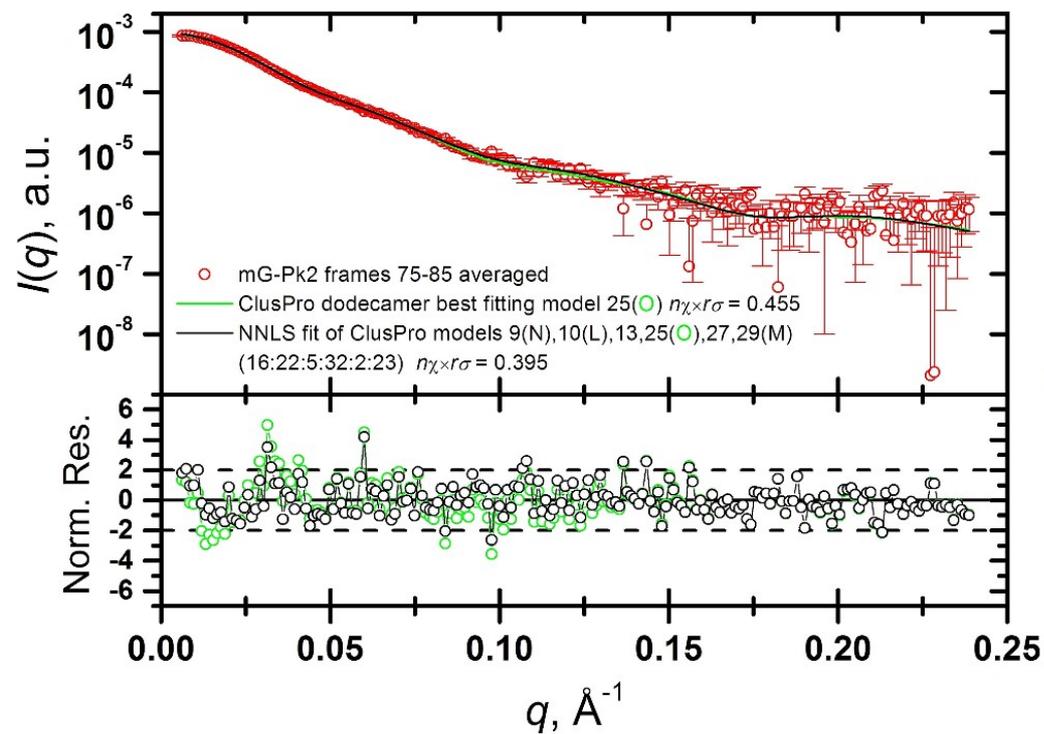
# Deconvolution of Aldolase & Model



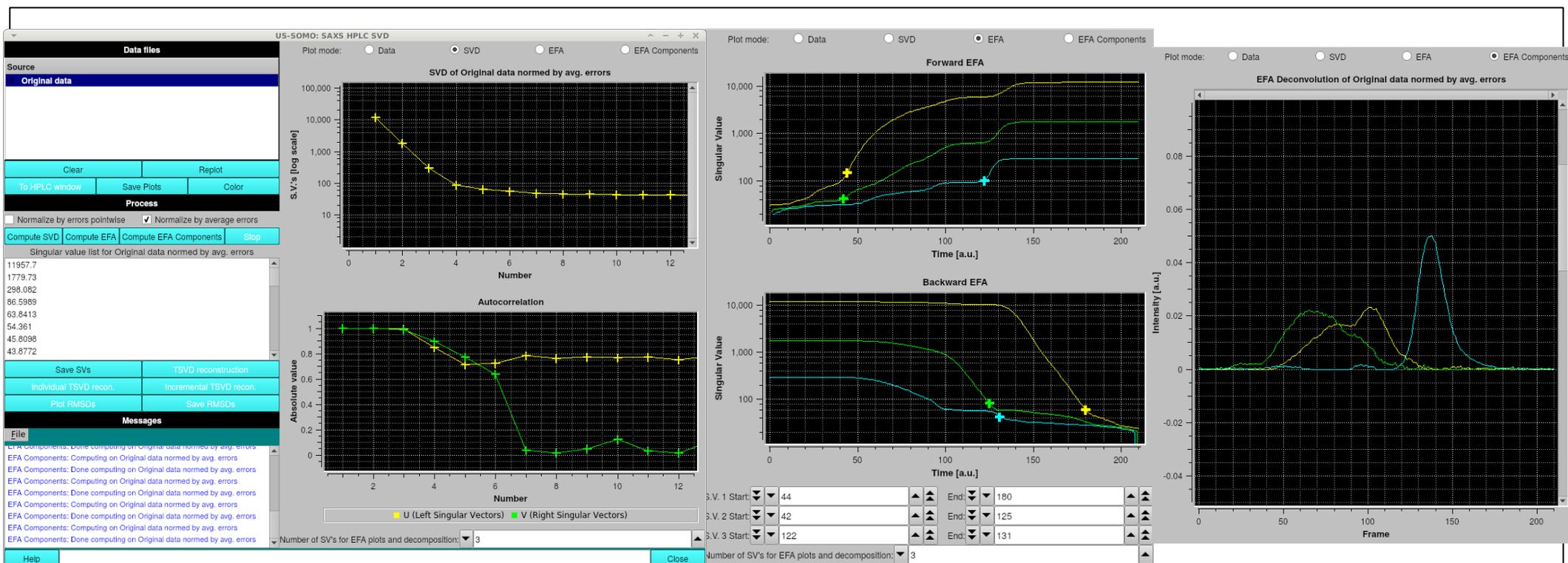
# Deconvolution of Aldolase & Model



# Deconvolution of Aldolase & Model



# SVD/EFA

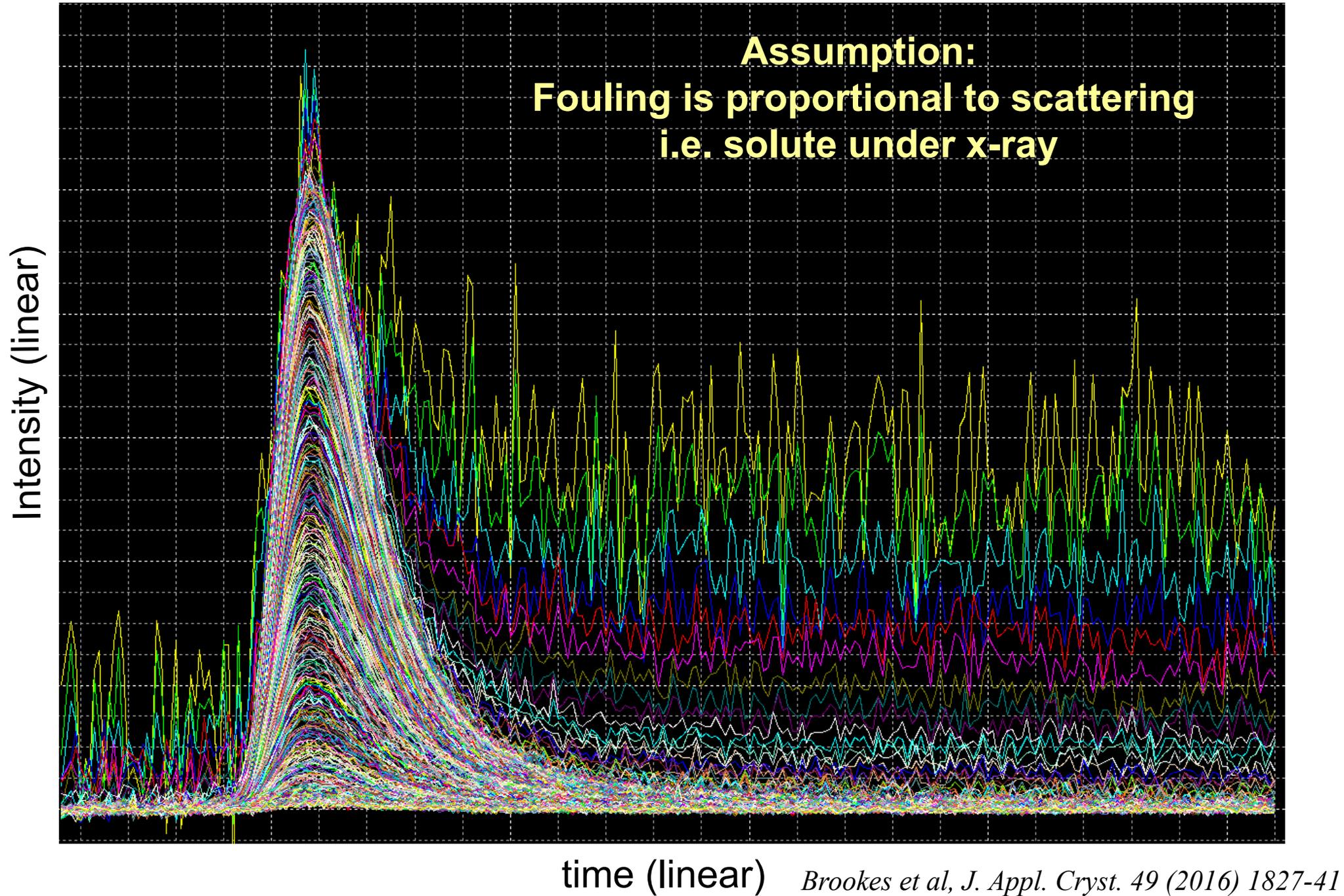


*Hopkins et al. (2017) J. Appl. Cryst. 50(5) 1545-53*

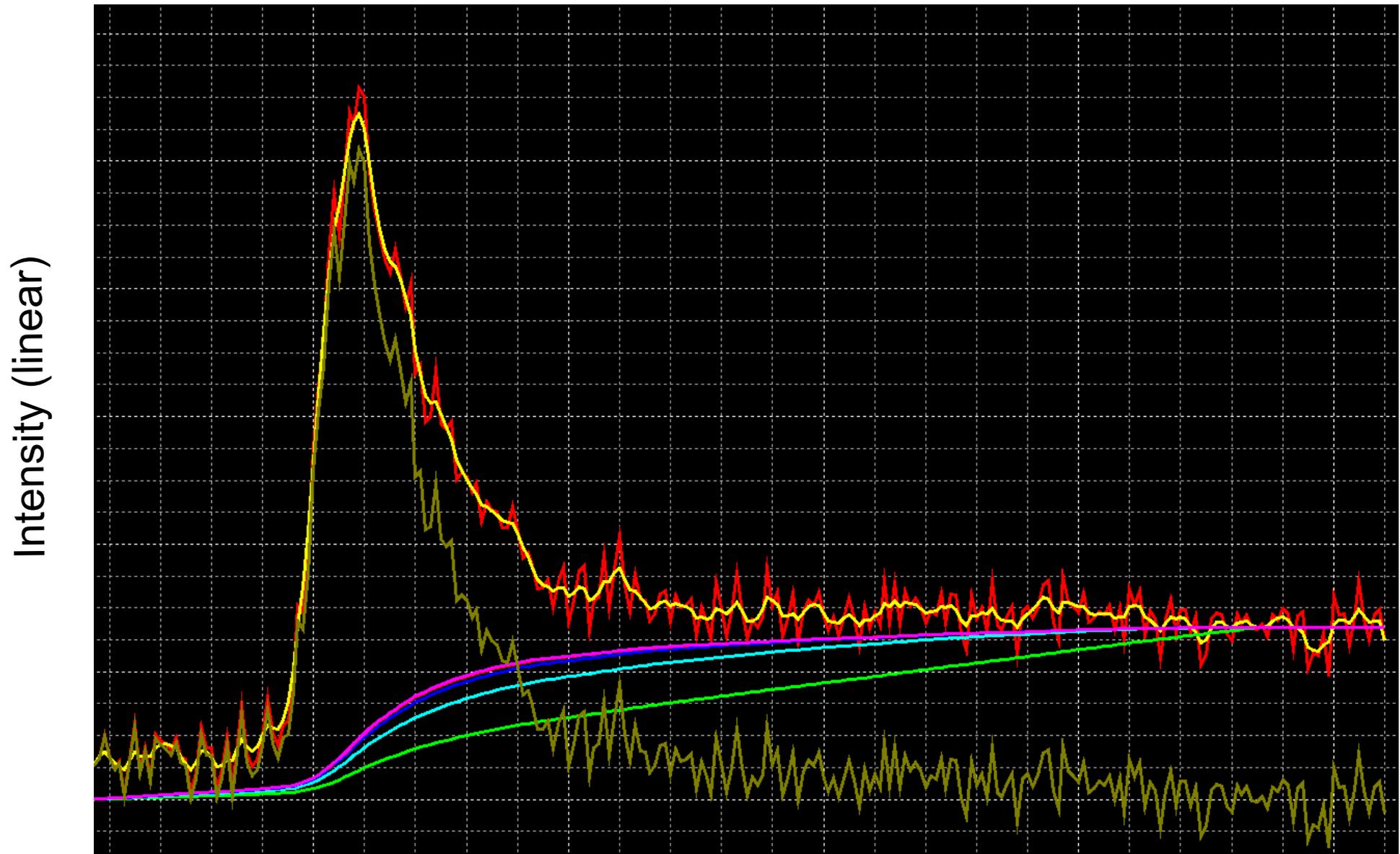
*Meisburger et al. (2016) J. Am. Chem. Soc. 138 6506-16*

*Brookes et al, J. Appl. Cryst. 49 (2016) 1827-41*

# SEC-SAXS $I(t)$ profiles Lyzosome with fouling



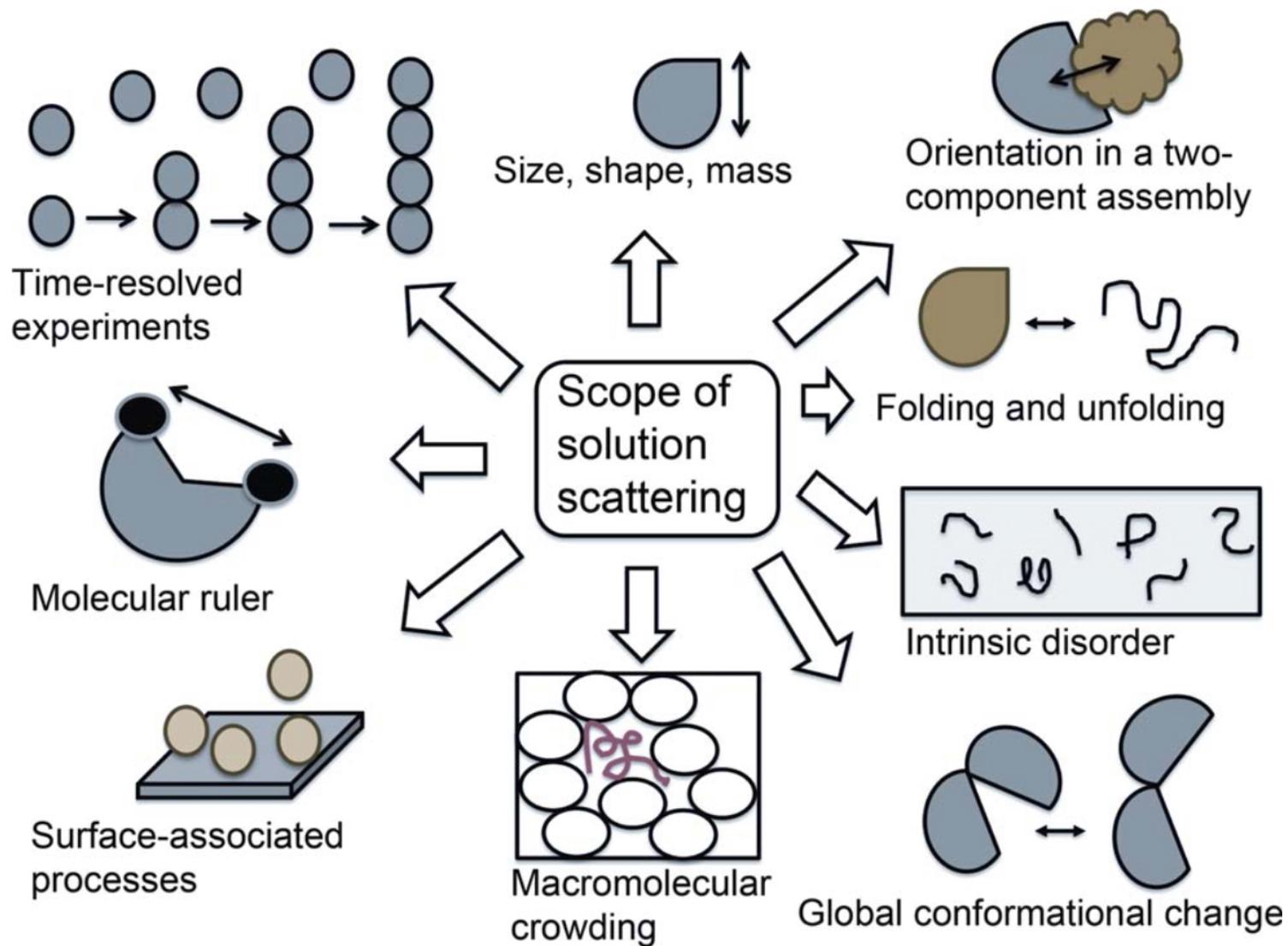
# $I(t)$ fouling correction



# SEC-SAXS

- If you have sufficient sample – use SEC-SAXS for biological macromolecules
- If you have true baseline separation, excellent, you should probably be ok simply taking the peak data, but global Gaussian decomposition will use all of the data
- If you do not have true baseline separation, be very careful and you should use these techniques

# Scope of solution scattering



# ***XFELs – SLAC/LCLS***



# XFELs

LCLS generating  $\sim 10^{12}$  photons per pulse @ 9 keV

Electron bunch ( $\sim 10^9$  electrons)

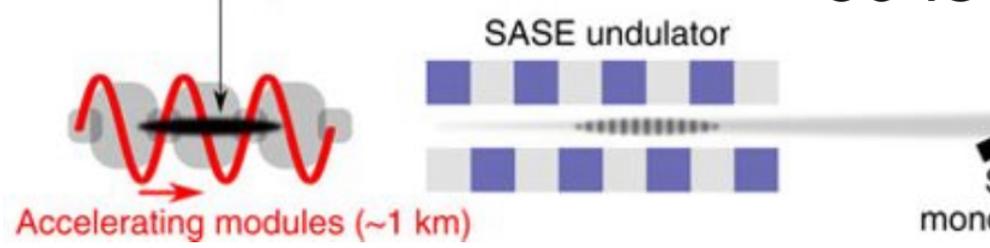
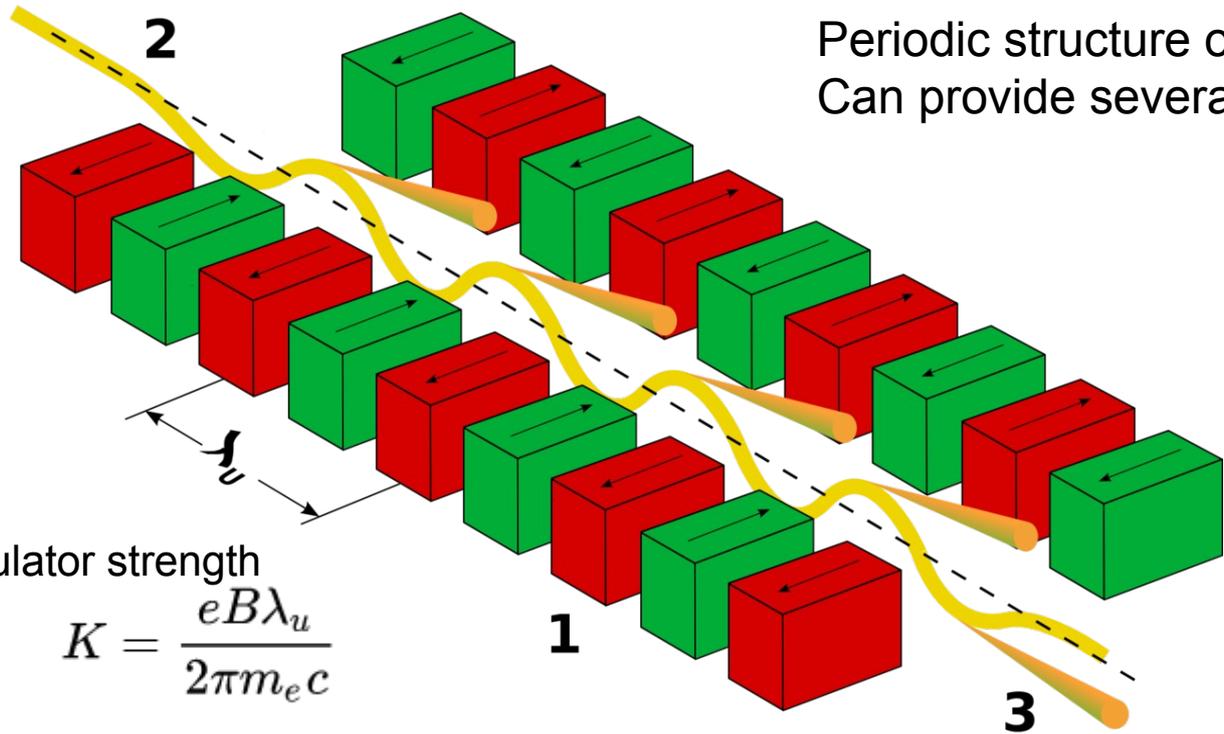


Image: Pearson

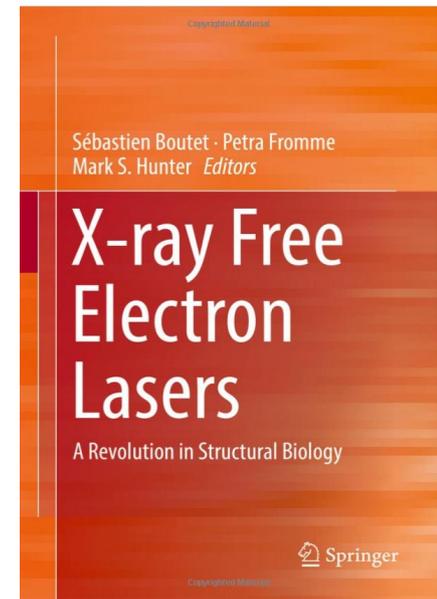
Periodic structure of dipole magnets  
Can provide several orders of magnitude higher flux



Undulator strength

$$K = \frac{eB\lambda_u}{2\pi m_e c}$$

CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=537945>



# XFEL SAXS/WAXS

2 point correlations :  $p(r) \rightarrow$  4 point correlations promises possibility of better 3d models

Still early days.

Few XFELs instruments

Serial access (vs synchrotron)



Elettra Sincrotrone Trieste

FERMI at ELETTRA



Flash at DESY



LCLS at SLAC



POHANG ACCELERATOR LABORATORY

PAL-XFEL

PAUL SCHERRER INSTITUT



Swiss FEL at PSI

# Schools



Beyond Rg Bio

<https://small-angle.aps.anl.gov/future-courses#BeyondRgBio>



National School on Neutron and X-Ray Scattering

<https://neutrons.ornl.gov/nxs>



Organized by Université Grenoble Alpes  
& Grenoble Institute of Technology



<http://hercules-school.eu/>



Practical courses

<http://embo.org/funding-awards/courses-workshops/practical-courses>

# Books

“La diffraction des rayons X aux très petits angles: Application a l’etude de phénomènes ultramicroscopiques”:

*A. Guinier (1939), Ann. de Phys., 11:12*

pdf in course papers

“Small Angle Scattering”:

*A. Guinier and A. Fournet, (1955), in English, ed. Wiley, NY*

“Small Angle X-Ray Scattering”:

*O. Glatter and O. Kratky (1982), Academic Press.*

pdf available

<http://physchem.kfunigraz.ac.at/sm/Software.htm>

“Structure Analysis by Small Angle X-ray and Neutron Scattering”:

*L.A. Feigin and D.I. Svergun (1987), Plenum Press.*

pdf available

[http://www.embl-hamburg.de/ExternalInfo/Research/Sax/reprints/feigin\\_svergun\\_1987.pdf](http://www.embl-hamburg.de/ExternalInfo/Research/Sax/reprints/feigin_svergun_1987.pdf)

“Neutrons, X-Rays and Light, Scattering methods applied to soft condensed matter”:

*P. Lindner and T. Zemb Eds, (2002) Elsevier, North-Holland.*

The Proceedings of the SAS Conferences held every three years are usually published in the Journal of Applied Crystallography